1. Use the QG equations to explain the large-scale circulation features of the Indian monsoon. Assume that the heat source due to latent heating is given by

\[
\frac{\partial}{\partial t} = \frac{\partial p}{\partial x} \sin(\frac{\pi}{2}) \cos(\frac{\pi}{2}) \cos(\frac{\pi}{2}) \quad \text{for} \ |x| \leq L, |y| \leq L,
\]

\[
\frac{\partial}{\partial t} = 0 \quad \text{for} \ |x| > L \text{ or } |y| > L
\]

a. Derive new terms in the \( \omega, \chi \) and QGPV equations which represent the heat source. Assume that the stability parameter \( \sigma \) is constant and \( f \approx f_0 \). To keep the algebra simple you can neglect the geostrophic advection terms in the vorticity and thermodynamic equations.

b. Find \( \omega \) for \( |x| \leq L, |y| \leq L \) and sketch the vertical profile of rising motion produced by the heating at \( x = y = 0 \).

c. Sketch the geopotential tendency at lower and upper levels (say 850mb and 150mb). Also, use the \( \odot \) and \( \otimes \) symbols to indicate the induced geostrophic winds, and indicate the regions of positive and negative divergence and PV tendency.

2. Suppose that the vorticity advection accompanying a baroclinic wave occurs entirely above the 300mb level, and produces a geopotential tendency \( \chi_{300} = A_0 \cos(kx) \) at 300mb.

a. Find the geopotential tendency induced at all levels below 300mb, assuming that there is no vorticity advection or thermal advection below that level. Assume that \( \sigma \) is constant.

To solve the problem, assume that \( \chi = A(p) \cos(kx) \) and write an ODE for \( A \) of the form

\[
\frac{d^2 A}{dp^2} = \lambda A
\]

Choose the solution for which the geopotential tendency at the surface is smaller than at 300mb.

b. How does the strength of the geopotential tendency at the surface depend on \( f_0, \sigma \), and the length scale \( L = 2\pi/k \)?