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Validation of a Tropical Cyclone Steering Response Function with a Barotropic Adjoint Model

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Abstract

The steering of a tropical cyclone (TC) vortex is commonly understood as the advection of the TC vortex by an “environmental wind.” In past studies, the environmental steering wind vector has been defined by the horizontal and vertical averaging of the horizontal winds in a box centered on the TC. The components of this environmental steering have been proposed as response functions to derive adjoint-derived sensitivities of TC zonal and meridional steering. The appropriateness of these response functions in adjoint sensitivity studies of TC steering is tested using a two-dimensional barotropic model and its adjoint for a 24hr forecast. It is found that these response functions do not produce sensitivities to TC steering, because perturbations to the model initial conditions that change the final-time location of the TC also change the response functions in ways that have nothing to do with the steering of the TC at model verification.

An alternate response function is proposed wherein the environmental steering vector is defined as the wind averaged over the response function box attributed to vorticity outside of that box. By redefining the response functions for the zonal and meridional steering as components of this environmental steering vector, the effect of small changes to the final-time location of the TC is removed, and the resultant sensitivity gradients can be shown to truly represent the sensitivity of TC steering to perturbations of the model forecast state.
1. Introduction

An adjoint sensitivity study involves evaluating the change in a specific aspect (called a response function, R) of a model forecast state \( x_f = x(t_f) \) resulting from arbitrary changes in any of the model control variables at the initial \( t = 0 \) or forecast times \( t = \tau \). Such a study calculates the gradient of the response function, \( \partial R / \partial x_\tau \), with respect to the model control variables represented by a state vector \( (x_\tau) \).\(^1\) An adjoint model is the most efficient tool for evaluating these forecast sensitivities (Errico 1997).

Conspicuously absent from the many prior applications of adjoint-based sensitivity analysis to tropical and extratropical cyclone issues are synoptic and dynamical interpretations of these sensitivities. When dynamical interpretation of sensitivities is provided, often what is offered is merely the observation that the distribution of sensitivities (Vukicevic and Raeder 1995, Wu et al. 2007) or singular vectors (Peng and Reynolds 2006, Chen et al. 2009) is coincident with a synoptic feature. Coincidence of adjoint sensitivities with a synoptic feature alone is insufficient to attribute dynamical significance to the feature (Langland et al. 1995). Langland et al. (1995) demonstrate that the sensitivity fields calculated in their study of an idealized cyclogenesis event in a time-evolving, non-zonal flow were very similar to the same calculation performed for a steady, zonal basic state. While adjoint sensitivities do not provide information regarding whether particular physical processes actually occurred within a basic state (control) forecast trajectory, these sensitivities do provide information concerning the effect

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\(^1\) Most often, sensitivities are calculated with respect to the model initial and boundary conditions; however, these dynamical sensitivities may be calculated with respect to the model forecast trajectory, \( x_\tau, 0 < \tau < t_f \), as well. For the purposes of this study, references to \( x \) can be inferred to mean \( x_f \) when referring to the response function, while references to \( x \) can be inferred to mean \( x_o \) when referring to sensitivities of the response function.
of possible perturbations to the basic state. It is this particular characteristic of adjoint sensitivities that make them a potentially powerful tool in synoptic case studies (Langland and Errico 1996).

In order for the results of an adjoint sensitivity calculation to have meaning, thorough testing and interpretation is required (Errico and Vukicevic 1992). Such rigorous dynamical interpretation and testing of adjoint-derived forecast sensitivities provides a means of characterizing what the sensitivity fields represent and evaluates how well the response function associated with the sensitivity gradients truly measures its intended forecast aspect. In this study, a specific set of response functions, designed to measure instantaneous TC steering, is considered. It is demonstrated that while the response functions chosen are appropriate for diagnosing steering, the sensitivities calculated with these response functions do not provide insight into the sensitivity of TC steering. A solution to this problem is offered and tested.

Wu et al. (2007) presents an objective targeting strategy whereby a response function is defined to represent the zonal or meridional steering of a TC at model verification:

\[ R_1 = \frac{\sum \sum u_{i,j,k} \Delta x \Delta y \Delta p}{\sum \sum \Delta x \Delta y \Delta p} \]  

(1)

\[ R_2 = \frac{\sum \sum v_{i,j,k} \Delta x \Delta y \Delta p}{\sum \sum \Delta x \Delta y \Delta p} \]  

(2)

\[ R_1 \] and \[ R_2 \] represent respectively, the horizontally and vertically averaged zonal and meridional wind in a horizontal domain \( D \) and bounded between 850 hPa and 300 hPa (indexed by \( k \)). The horizontal domain \( D \), or the “response function box”, is defined as the set of all grid points (indexed zonally by \( i \) and meridionally by \( j \)) in a box 600 km on a side, centered on the model...
verification position of the TC. In this way, the response function $R_1$ ($R_2$) is the environmental zonal (meridional) wind steering the TC at verification (Chan and Gray 1982). The sensitivity gradients can be combined into a vector:

$$ADSSV = \begin{pmatrix} \frac{\partial R_1}{\partial x} \\ \frac{\partial R_2}{\partial x} \end{pmatrix}$$

that represents the vector change in steering of the TC given a unit perturbation to a model state vector, $x$. Wu et al. (2007) calls these vectors “Adjoint-Derived Sensitivity Steering Vectors”, or ADSSVs, and uses the vectors in an attempt to understand the influence of dropsondes deployed in a Dropwindsonde Observations for Typhoon Surveillance near the Taiwan Region (DOTSTAR) targeted observation field campaign (Wu et al. 2005) to initialize model forecasts of Typhoon Mindulle (2004). Wu et al. (2007) suggests that because these dropsondes provided data in regions of low sensitivity their assimilation would not have significantly improved track forecasts for Mindulle.

While care has been taken to show that sensitivities using these TC steering response functions are coincident with synoptic features likely important to the steering of a modeled TC (Wu et al. 2009), this coincidence is insufficient to identify those synoptic features as important. Further, no studies exist that test the validity of these response functions by perturbing the initial conditions of the model in regions of high sensitivity, and determining exactly how those perturbations impact the response function. Due to the lack of any rigorous testing of the appropriateness of these response functions and a lack of dynamical interpretation of the sensitivity fields, it is unclear whether these sensitivity gradients appropriately describe sensitivity to TC steering.

Using a 2-D, barotropic, non-divergent, inviscid model on an $f$-plane (see Section 2), it is shown in this study that the response functions previously used to define TC steering (see Wu et
al. 2007, Wu et al. 2009) are inappropriate for diagnosing TC steering sensitivity. A dynamical interpretation of the sensitivity gradients for $R_1$ and $R_2$ in the simplified model provides an explanation of the problem as well as the solution. Section 2 provides a description of the model and the idealized case used in the study. A dynamical interpretation of the sensitivities is provided in Section 3, along with an analysis of perturbations introduced into the model in order to verify the interpretation. In Section 4, a solution to this problem is tested and verified by redefining the response functions used by Wu et al. (2007) to describe TC steering. Future work using these tools is described in Section 5.

2. Model Study

a) The Model

The simplified numerical model used is based upon the two-dimensional, non-divergent inviscid barotropic vorticity equation on an $f$-plane, for which relative vorticity is conserved:

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla \zeta = 0 \quad (4),$$

where $\zeta$ is the relative vorticity and $\mathbf{V}$ is the horizontal vector wind field with zonal and meridional components $u$ and $v$ respectively. A streamfunction, $\psi$, which is related to the relative vorticity by $\nabla^2 \psi = \zeta$, is diagnosed from the vorticity distribution using successive over-relaxation. The non-divergent wind field is calculated from the streamfunction:

$$u = -\frac{\partial \psi}{\partial y} \quad (5)$$

$$v = \frac{\partial \psi}{\partial x} \quad (6).$$

The nonlinear model is solved on a centered-difference, discretized $f$-plane, over a closed domain 5235 km across with a grid spacing of 15 km. Homogeneous lateral boundary conditions
are imposed on the streamfunction. The model is integrated forward in time using a forward Euler scheme, with a 30s time-step.

b) The tangent-linear and adjoint models

Linearization of (4) about a time-evolving forecast trajectory calculated from (4) yields the tangent linear model:

$$\frac{\partial \zeta'}{\partial t} + \nabla \cdot \nabla \zeta' + V' \cdot \nabla \zeta' = 0$$ (7)

where \(\overline{\zeta}\) and \(\overline{V}\) are the basic state relative vorticity and wind vector \(\overline{V} = (\overline{u}, \overline{v})\) respectively, and \(\zeta'\) and \(V'\) are the perturbation relative vorticity and wind vector \(V' = (u', v')\) respectively. The tangent linear model is also solved using a centered in space, forward Euler in time scheme. The adjoint model is developed at the coding level as the line-by-line transpose of the tangent linear model. Tests of the validity and accuracy of the tangent linear and adjoint codes are performed in sections 3b and 4c and d. Because the dynamics of the model are expressed in terms of the distribution of relative vorticity, output of the adjoint model is sensitivity with respect to vorticity.

c) Idealized Case

In order to aid dynamical interpretation of adjoint-derived sensitivities of TC steering, the simplest possible idealization was chosen – a TC embedded in a quiescent environment. The nonlinear model was initialized with a Gaussian distribution of vorticity (hereafter referred to as the basic state vortex, or BSV) maximized at the center of the model domain (Fig. 1). This vortex is the “TC” in the model. The model is run for 24 hours. Since the only wind in the model domain is symmetric about the BSV center, the state of the model is unchanged for the

\[\text{\footnotesize\textsuperscript{2}}\text{ The code and supporting documentation are available from the first author upon request.}\]
full 24-hour integration. This 24-hour model trajectory defines the basic-state about which the adjoint model is linearized.

The response function is defined as the average zonal flow in a 1200 km x 1200 km box centered on the BSV at the end of the 24-hour model integration

\[ R_{i} = \frac{\sum_{i,j \in D} u_{i,j} \Delta x \Delta y}{\sum_{i,j \in D} \Delta x \Delta y} \]  

(8).

For brevity, the discussion to follow focuses on response functions for zonal steering.

3. Description and interpretation of \( \partial R_{i} / \partial \zeta \)

a) Dynamics

Before describing the sensitivities of \( R_{1} \) to vorticity, we consider what perturbations to the model forecast trajectory would change \( R_{1} \), the average zonal flow in the response function box at forecast hour 24 given the dynamical constraints of the model. The only way to increase \( R_{1} \) at 24 hours is for positive (negative) relative vorticity perturbations to be found north (south) of the center latitude of the box at that time. Within the context of the linearized model, there are only two processes which can create this state: 1) the basic state flow advects positive (negative) perturbation vorticity north (south) of the BSV position; or 2) the creation of vorticity perturbations associated with the northward advection of the BSV (translation of the TC) by winds associated with vorticity perturbations external to the BSV. The former effect, the basic state advection of perturbation vorticity (second term in the linearized model (7)), is the instantaneous steering effect we seek to capture using the adjoint sensitivity diagnosis. The latter effect, an increase in \( R_{1} \) due to a northward displacement of the BSV (manifest as an asymmetric distribution of perturbation vorticity in the box, third term in (7)) is not a zonal steering effect.
Further, this latter effect represents the integrated response of the imposed vorticity perturbations west and east of the longitude of the BSV advecting that vortex northward rather than the instantaneous response associated with the north-south perturbation vortex dipole.

b) Adjoint sensitivity gradients for $R_1$

The adjoint model is initialized with the distribution of $\partial R_1 / \partial \zeta$ at the end of the 24-hour model integration (Fig. 1a). These sensitivities describe how a perturbation to vorticity would instantaneously change the response function. Sensitivities at this time are positive (negative) to the north (south) of the BSV (Fig. 1a). This distribution of sensitivities implies that placing positive (negative) vorticity perturbations north (south) of the response function box increases $R_1$. Such perturbations correspond to an increase in the zonal flow that would steer the BSV (the “TC”) eastward at this time.

Integrated backward in time 24h to the model initialization time, $\partial R_1 / \partial \zeta$ has two important features (Fig. 1c). First, the maximum positive (negative) sensitivity appears northeast (southwest) of the BSV center. Second, positive (negative) sensitivity now appears west (east) of the BSV center. Each of these two features can be identified with processes in (7): the advection of perturbation vorticity by the basic state wind field (the instantaneous steering effect) and the advection of the basic state vorticity by the perturbation wind field (the primary cyclone displacement effect).

c) Interpretation

1) INSTANTANEOUS STEERING EFFECT

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$^3 \partial R_1 / \partial \zeta$ is computed by defining $\partial R_1 / \partial u$, $\partial R_1 / \partial v$ directly and computing $\partial R_1 / \partial \zeta$ by using the adjoint of the successive over-relaxation scheme used to compute $u$, $v$ from $\zeta$.  

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Sensitivity of $R_1$ with respect to vorticity at 24 hours is a north-south dipole of positive and negative vorticity centered on the BSV. Because the absolute (and relative) vorticity is conserved on an $f$-plane in the model, at forecast hours prior to 24 hours these vorticity perturbations must be located upstream of their final-time locations (Fig. 2a). How far upstream the vorticity perturbations must be located to maximize their influence on increasing $R_1$ is dependent on the length of the adjoint model backward integration, the distance the vorticity perturbation is from the center of the box, and the intensity of the BSV.

Perturbation vorticity northeast and southwest of the BSV at the forecast hour 0 will be advected by the basic state wind field (Fig. 2a). At the end of the 24-hour model integration, this perturbation vorticity will be directly north and south of the BSV center (Fig. 2b). Therefore, if positive (negative) perturbation vorticity were introduced northeast (southwest) of the BSV at model initialization, the dipole of positive and negative perturbation vorticity would be oriented north/south across the response function box by the end of the model integration. This perturbation vorticity would be associated with a positive zonal perturbation flow in the response function box, increasing $R_1$.

2) PRIMARY CYCLONE DISPLACEMENT EFFECT

Unlike the instantaneous steering effect, the distribution of perturbation vorticity which would create a northward displacement of the BSV is less obvious because of the dual (possibly competing) effects of the imposed vortices – the vortices will not only perturb the location of the BSV, they will also directly contribute to the zonal flow in the box at the time the response function is defined.

In order for perturbation vorticity to advect the BSV northward, the positive (negative) perturbations must be located northwest (southeast) of the longitude of the BSV at some time
over the 24h period. In this configuration, the perturbation flow would be from the south over the BSV. Positive (negative) perturbation vorticity placed initially to the west (east) of the BSV would be associated with a southerly perturbation flow, advecting the BSV to the northeast (Fig. 2c). By the end of the 24-hour model integration, the BSV is displaced northeast of the center of the response function box (Fig. 2d). The symmetric circulation of the BSV contributes positively to the average zonal flow in the response function box, positively influencing $R_1$.

The dynamical interpretation of the adjoint-derived sensitivity gradients makes the problem with the $R_1$ clear: the process described by the basic state advection of perturbation vorticity actually impacts the steering of the BSV. Sensitivities associated with this process can legitimately be called sensitivities of TC steering. The sensitivities associated with perturbation advection of basic state vorticity are of another sort. Perturbations to the model initial conditions in regions where these sensitivities are large result in the BSV being advected north of its final-time location in the control forecast and therefore contribute to $R_1$ in a way that has nothing to do with the zonal steering of the BSV. We call this the primary cyclone displacement (PCD) effect.

While it would be convenient to simply move the averaging box so as to be centered over the new vortex center and thus remove the effect of the displaced cyclone, the response function is chosen as the average zonal wind in a box over a specific geographic region. While the chosen region may be centered directly on the vortex in the basic state, and therefore represent the steering of a modeled TC in the basic state, there is nothing preventing vorticity perturbations from advecting the vortex out of the middle of the box. Any comparison, qualitative or quantitative, between these sensitivities and the steering of the modeled TC must keep the averaging box in the same place. The response function box cannot be arbitrarily moved based on a posteriori information in order to accommodate the perturbation advection of the BSV.
d) **Perturbation test for validation of dynamical interpretation**

A test can be performed to validate the above dynamical interpretation for $\partial R_1/\partial \zeta$. Initial vorticity perturbations, designed to increase $R_1$, may be derived by scaling the sensitivity to the initial distribution of vorticity, $\frac{\partial R_1}{\partial \zeta} \bigg|_{\tau=0h}$:

$$\zeta'_0 = S \frac{\partial R_1}{\partial \zeta} \bigg|_{\tau=0h} \max \left\{ \left| \frac{\partial R_1}{\partial \zeta} \right|_{\tau=0h} \right\}$$

Here, $\max \left\{ \left| \frac{\partial R_1}{\partial \zeta} \right|_{\tau=0h} \right\}$ is the absolute value of the maximum sensitivity in the model domain at forecast time 0h. An initial perturbation vorticity distribution with maximum perturbation vorticity $2.0 \times 10^{-6}$ s$^{-1}$ is created by choosing $S = 2.0 \times 10^{-6}$. In this way, perturbation vorticity introduced at model initialization has the same structure as $\frac{\partial R_1}{\partial \zeta} \bigg|_{\tau=0h}$ (Fig. 1c).

The model is integrated forward 24h with the perturbed initial conditions. At 24 hours, perturbation vorticity and winds, defined as the difference between the perturbed and control (unperturbed) forecast values of vorticity and wind respectively, reveal a dipole of perturbation vorticity in the response function box indicating a northeast translation of the BSV by the perturbation wind field (Fig. 1e). A significant portion$^4$ of the perturbation flow in the response function box is attributed to the vorticity perturbations ascribed to the translation of the BSV. This result confirms the dynamical interpretation for the fault with $\partial R_1/\partial \zeta$ - that the assumption that the BSV would remain in the center of the response function box is violated, and some contributions to the change in $R_1$ have nothing to do with the instantaneous zonal steering of the

$^4$ Perturbation zonal flow attributed to the northward displacement of the BSV accounts for 55% of the perturbation zonal flow in the response function box.
BSV at 24 hours. In fact, it appears that, for this case, the PCD effect is larger than the instantaneous steering effect.

We can also perform a test to validate the adjoint model’s assumption of linearity for this case. The total change in the response function between the perturbed and control runs can be directly calculated by evaluating $R_1$ for each run and calculating the difference.

$$\Delta R_1 = R_1|_{perturbed} - R_1|_{control}$$  \hspace{1cm} (10)

Assuming linearity, this value can be approximated by evaluating the inner product of the sensitivity field at model initialization $\frac{\partial R_1}{\partial \zeta} = 0$ with the perturbation to initial condition vorticity $\zeta' = 0$.

$$\Delta R_1 \cong \delta R_1 = \left( \frac{\partial R_1}{\partial \zeta} |_{\zeta = 0} , \zeta' = 0 \right)$$  \hspace{1cm} (11)

For the case under consideration, $\Delta R_1 = 0.922\text{ m s}^{-1}$, while $\delta R_1 = 0.919\text{ m s}^{-1}$. The adjoint model was able to account for 99.7% of the change in $R_1$, indicating that the perturbations evolved linearly in the model, and the adjoint-derived sensitivity gradients are valid.

As Fig. 1e shows, the change in $R_1$ is largely the result of a slight northward displacement of the final-time location of the BSV within the response function box. This does not correspond to an increase in the zonal steering of the BSV at this time. The PCD effect does not appear to be dependent on the physical size of the BSV (not shown).

The strength of the PCD effect is partially dependent on the size of the response function box. The PCD effect is manifest as a dipole of perturbation vorticity in the center of the box with a concomitant perturbation flow pattern described by that vorticity. While a larger response function box would do nothing to reduce the size of this dipole, the contribution of the PCD effect on the average perturbation flow would be smaller for a large response function box than
for a small box. However, this cannot serve as a solution to this problem. The PCD effect exists regardless of the size of the response function box. And the box cannot be too large, since the response function is intended to describe the environmental flow in the vicinity of the TC.

In the following section, a new response function is introduced in order to provide a solution to this problem, and is tested in the same manner as $R_1$.

4. Proposed solution

a) Environmental steering response function

It has been shown that the response function used to describe TC steering is fundamentally flawed. Perturbations to the final-time location of the TC within the response function box allow for the TC’s own symmetric circulation to contribute to $R_1$ (and $R_2$) in a way that has nothing to do with the instantaneous steering of the TC. Dynamical interpretation of $\partial R_1 / \partial \zeta$ shows that the PCD effect is manifest as positive (negative) sensitivity with respect to vorticity appearing west (east) of the BSV.

Here, we propose new response functions to define the zonal and meridional steering of a TC, such that the PCD effect is removed from the sensitivity gradients associated with the new response functions. For a given domain, one can define the “hurricane advection flow” as the balanced flow at the TC center attributed to the potential vorticity (PV) within that domain after the PV of the TC has been removed (Wu and Emanuel, 1995).

For two-dimensional, non-divergent, barotropic flow, the relevant potential vorticity is the absolute vorticity. We designate the vorticity within the response function box ($D$) as vorticity of the BSV, and all of the vorticity outside of the response function box as vorticity of the environment. The partitioning of the full domain vorticity into vorticity of the TC (vorticity
in box) and vorticity of the environment is accomplished by the use of a linear environmental projection operator, \( E \):

\[
E \zeta_{i,j} = 0 \text{ if } i,j \in D \text{ and } E \zeta_{i,j} = \zeta_{i,j} \text{ if } i,j \notin D \quad (12),
\]

where \( \zeta_{i,j} \) represents the grid point representation of the vorticity.

Once the vorticity of the TC is removed, the vorticity of the environment can be inverted to recover the “environmental” wind, \( \tilde{V} : \tilde{V} = Q^{-1}(E \zeta) = k \times \nabla [\nabla^{-2}(E \zeta)] = \tilde{u}i + \tilde{v}j \), where \( Q \) is an operator that calculates the two-dimensional, non-divergent wind field from the vorticity distribution and “\( \nabla^{-2} \)” represents the successive over-relaxation inversion operator that calculates streamfunction from vorticity.

We define a new set of response functions to describe the average “environmental” zonal and meridional wind in the vicinity of the TC.

\[
R_{E1} = \frac{\sum_{i,j \in D} \tilde{u}_{i,j} \Delta x \Delta y}{\sum_{i,j \in D} \Delta x \Delta y} \quad (13)
\]

\[
R_{E2} = \frac{\sum_{i,j \in D} \tilde{v}_{i,j} \Delta x \Delta y}{\sum_{i,j \in D} \Delta x \Delta y} \quad (14)
\]

The averaging of the environmental wind \( (\tilde{u}, \tilde{v}) \) is performed in the same manner as the averaging of the full wind in \( R_1 \) and \( R_2 \). Figure 3a is a flowchart describing the procedure used to calculate \( R_{E1} \) and \( R_{E2} \) given the model state as expressed in terms of vorticity. The adjoint of this procedure, shown in Fig. 3b, is used to derive the sensitivity gradients that initialize the adjoint model.
Since all vorticity within the response function box is removed when the environmental wind is calculated, small perturbations to the final-time location of the BSV within the response function box have no effect on $R_{E1}$ and $R_{E2}$. A translation of the BSV caused by advection by the perturbation wind field is manifest as a dipole of positive and negative perturbation vorticity oriented in the direction of the translation (Fig. 1e). As long as the vorticity associated with the TC is not advected outside of the response function box, any translation of the TC within the box has no effect on the response functions. This assumption is less restrictive than the previous assumption that the TC had to remain in the center of the response function box.

b) Adjoint sensitivity gradients for $R_{E1}$

By construction, the distribution of $\partial R_{E1}/\partial \zeta$ that is used to initialize the adjoint model (Fig. 1b) is identical to the initialized sensitivity of $R_1$ (Fig. 1a), except that sensitivity of $R_{E1}$ to vorticity is zero within the response function box. Since all vorticity within the response function box is removed in order to calculate $\tilde{u}$, perturbations to vorticity within the response function box can have no effect on $R_{E1}$ at this time.

The structure of $\partial R_{E1}/\partial \zeta$ at time $t = 0$ (Fig. 1d) looks quite different from that of $R_1$ (Fig. 1c). Maximum sensitivities still appear to the northeast and southwest of the BSV, but the sensitivities west and east of the BSV associated with the PCD effect have vanished. In fact, there is weak sensitivity of the opposite sign present west and east of the BSV, compared to the sensitivity of $R_1$. The differences in the $\partial R_1/\partial \zeta$ and $\partial R_{E1}/\partial \zeta$ fields at adjoint model initialization (Figs. 1c and 1d respectively) appear primarily directly west and east of the BSV (Fig. 4a), and are related to the PCD effect.

c) Perturbation test for validation of dynamical interpretation

A test similar to that performed for the dynamical interpretation of $\frac{\partial R_1}{\partial \zeta}$ (section 3b) is performed here for dynamical interpretation of $\frac{\partial R_{E1}}{\partial \zeta}$. The sensitivity gradient of $R_{E1}$ with respect to vorticity is scaled to create an initial condition perturbation vorticity field that will increase $R_{E1}$. The nonlinear model is then integrated forward 24 hours with the perturbed initial conditions.

The perturbation environmental vorticity and perturbation environmental wind at 24 hours reveal that the perturbations to increase $R_{E1}$ create a uniform zonal perturbation environmental flow in the response function box (Fig. 1f). After the perturbation vorticity within the response function box is removed, the vorticity that remains (constituting the “environmental” vorticity) appears as a positive (negative) gyre directly north (south) of the response function box. Inversion of this environmental vorticity yields a zonal perturbation environmental wind in the response function box. Thus, the zonal steering of the TC, defined as the “hurricane advection flow” due to vorticity outside of the response function box, has been increased.

A test of linearity similar to that performed for the previous perturbation case is performed. For this case, $\Delta R_{E1} = 0.416 \text{ m s}^{-1}$, and the approximation of this value assuming linearity is $\delta R_{E1} = 0.420 \text{ m s}^{-1}$, indicating that the adjoint model accounts for 99.0% of the change in $R_{E1}$.

\textit{d) Elimination of PCD effect in $R_{E1}$}

The reversal in sign between $\frac{\partial R_1}{\partial \zeta}$ and $\frac{\partial R_{E1}}{\partial \zeta}$ in these regions west and east of the BSV indicates that positive (negative) perturbation vorticity introduced west (east) of the BSV at model initialization will increase $R_1$ and decrease $R_{E1}$. To show that the PCD effect is eliminated from $\frac{\partial R_{E1}}{\partial \zeta}$, another perturbation run is performed with a positive (negative)
perturbation vortex introduced west (east) of the BSV at the initialization of the nonlinear forward model (Fig. 4b). These perturbation vortices have maximum amplitude of $4.5 \times 10^{-6} \text{s}^{-1}$. Perturbation vorticity and winds are calculated at the end of the 24-hour model integration, and the response functions $R_1$ (Fig. 4c) and $R_{E1}$ (Fig. 4d) are evaluated.

Perturbation vorticity and winds contributing to $R_1$ (Fig. 4c) show a definite PCD effect. The dipole in perturbation vorticity in the response function box is a clear indication that the BSV has been advected to the northwest by the perturbation wind field by this time. Likewise, the perturbation vortices introduced at model initialization have been advected cyclonically around the BSV by the basic state wind field, appearing to the southwest and northeast of the response function box. The average perturbation zonal wind in the response function box is positive, with $\Delta R_1 = 0.101 \text{ m s}^{-1}$, while $\delta R_1 = 0.101 \text{ m s}^{-1}$, with the adjoint model accounting for 99.6% of the change in $R_1$.

By removing any perturbation vorticity in the response function box, the perturbation environmental vorticity and perturbation environmental winds contributing to $R_{E1}$ (Fig. 4d) reveals a reversal in the direction of zonal flow in the box. All significant perturbation vorticity associated with the PCD effect existed within the response function box. Since all perturbation vorticity in the response function box is removed in order to calculate $R_{E1}$, the PCD effect has no influence on the calculation of $R_{E1}$. The perturbation vorticity on the southwest and northeast corners of the response function box constitutes the “environmental” vorticity. This environmental vorticity induces an “environmental” zonal wind which is negative in the response function box, with $\Delta R_{E1} = -0.141 \text{ m s}^{-1}$. The approximation of $\Delta R_{E1}$ assuming linearity is $\delta R_{E1} = -0.138 \text{ m s}^{-1}$, indicating that the adjoint model could account for 97.9% of the change in $R_{E1}$. 

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This analysis validates the dynamical interpretation of $\partial R_1 / \partial \zeta$ and $\partial R_{E1} / \partial \zeta$ fields, and how those differences relate to the PCD effect. Sensitivities of $R_1$ west and east of the BSV are of opposite sign compared to sensitivities of $R_{E1}$ because for the perturbations considered, the PCD effect introduces *positive* perturbation flow in the response function box, while the vorticity that constitutes the “environment” introduces *negative* perturbation flow in the response function box. The PCD effect dwarfs the effect of the environmental vorticity, making $\Delta R_1$ positive. Since the PCD effect is eliminated in the calculation of $R_{E1}$, only the influence of the environmental vorticity remains, making $\Delta R_{E1}$ negative.

This methodology comes with its own limitations. The response function box separates vorticity associated with the TC (vorticity inside the box) from vorticity associated with the environment (vorticity outside the box). Any vorticity of an environmental feature that migrates into the box will be zeroed-out in the response function and not contribute to the “hurricane advection flow” assumed to steer the TC. Thus any interaction of the TC with its environment that produces environmental vorticity inside of the response function box at model verification will not be manifest in the sensitivities.

**5. Conclusions**

It has been shown that the definition of TC steering as a deep layer mean wind averaged around a TC center is not suitable for defining response functions to calculate adjoint-derived sensitivities of TC steering. The problem is that the deep-layer mean wind averaged around the TC is a measure of the environmental flow that steers the TC only so long as the TC is in the center of the averaging box, such that the symmetric circulation about the TC is removed in the averaging. This definition has been validated when calculating the steering flow for both observed and modeled TCs (Chan 2005). However, when the calculation is used to define a
response function for the purpose of calculating adjoint-derived forecast sensitivities of TC steering, the assumption that the TC remains in the same location is often violated. Since the response functions \( R_1 \) and \( R_2 \) can be influenced by perturbing either the environmental flow in the vicinity of the TC (a change to the steering) or the final time location of the TC within the response function box (not a change to the steering), these sensitivity gradients do not necessarily correspond to sensitivities of instantaneous TC steering.

A solution is proposed and validated in the context of a two-dimensional, non-divergent barotropic model by defining new response functions that eliminate the PCD effect. By defining the steering of the TC by the “hurricane advection flow”, which is an average of the environmental flow in the vicinity of the TC rather than an average of the full flow, the PCD effect is removed. While \( R_1 \) and \( R_2 \) require that the TC remain in the center of the response function box, the new response functions \( R_{E1} \) and \( R_{E2} \) only require that the vorticity associated with the TC remain within the response function box.

While the solution presented here has been validated within the context of a barotropic model, the \( R_{E1} \) and \( R_{E2} \) response functions should be tested in real TC simulations using a full-physics NWP model and that model’s adjoint. Clearly, issues regarding the validity of the assumption of linearity in such a model will include issues of the effect of diabatic processes that are not present in the simple model. While the relationship between the adjoint-derived sensitivity gradients and the physics of the tangent-linear model cannot be as clear in a full-physics model as they are in the simple model, it is expected that the dynamical interpretations of sensitivity gradients provided in this study will be a helpful corollary in a more complicated adjoint model. Real TCs are steered by a complexly-evolving environment that can itself be sensitive to small changes to the initial conditions; in such a case one would expect the steering
flow of the TC to be sensitive to the TC vortex as well as to relevant aspects of the environment. The results of this ongoing work will be provided in a future study.

A future application of this methodology includes the use of these adjoint-derived sensitivities to define targets for targeted observations with the explicit goal of improving TC track prediction. Dynamical sensitivities of TC steering to model initial conditions provide the requisite *a priori* information about where perturbations to the initial conditions of an NWP model will have the strongest effect on TC steering specifically. In order to apply this technique in an operational context, it would be necessary to determine an appropriate response function box size. The box needs to be large enough to accommodate the migration of the TC as a result of perturbations to the initial conditions, but small enough to produce meaningful results.

Dynamical sensitivities alone are insufficient to make well-informed choices about adaptive observation targeting. Dynamical sensitivity must be combined with information about the uncertainty in the initial conditions and information about how a given observation would be assimilated into the initial conditions. Coupling the output of an adjoint model with the adjoint of a data assimilation system would allow for the calculation of sensitivities of a response function to (individual) observations (Langland and Baker, 2004). By combining *a priori* information about the dynamical sensitivity to the model state, the uncertainty in the initial conditions, and the impact of a given observation assimilated into the initial conditions, sensitivities of an appropriate response function for TC steering can be utilized in a truly objective targeting strategy to improve TC track forecasting.

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References


Figure 1. Basic state vorticity contoured every $2.0 \times 10^{-5}$ s$^{-1}$ and sensitivity of $R_1$ with respect to vorticity shaded every $5.0 \times 10^{4}$ m at model verification (a) and model initialization (c). The same plots are provided for sensitivity of $R_{E1}$ with respect to vorticity at model verification (b) and model initialization (d). (e) Perturbation vorticity shaded every $5.0 \times 10^{-6}$ s$^{-1}$ and perturbation winds at model verification after optimal perturbations to increase $R_1$ were introduced at model initialization. (f) Perturbation environmental vorticity shaded every $5.0 \times 10^{-7}$ s$^{-1}$ and perturbation environmental winds at model verification after optimal perturbations to increase $R_{E1}$ were introduced at model initialization. Vectors in (f) are an order of magnitude smaller than vectors in (e). The box at the center of the plot corresponds to the boundaries of the response function box. Note that the wavy patterns in sensitivity in panel d are due to the zeroth-order discontinuity in initialized sensitivity (see panel b), and do not negatively impact the results.
Figure 2. (a) Schematic configuration of vorticity perturbations (‘+’ for positive perturbation, ‘-’ for negative) at initial forward model time placed in regions of maximum sensitivity shown in Fig. 1c; (b) subsequent configuration of vorticity perturbations 24h later that contribute to the *instantaneous* zonal flow within the response function box; (c) configuration of vorticity perturbations contributing to a northeastward “primary cyclone displacement” effect at the forward model initial time; and (d) subsequent configuration of the BSV and vorticity perturbations 24h after the time shown in (c). Note that in (d) the BSV has moved north-northeast of its initial location. Tropical cyclone symbol denotes location of BSV. Black filled arrows indicated flow associated with BSV. Grey filled arrows show direction of flow at vortex center attributed to perturbation vorticity. Shading indicates approximate regions of maximum sensitivity for $R_1$ that would produce the steering effect (a) and (b) and the PCD effect (c).
Figure 3. (a) Flow chart describing how the response functions $R_{E1}$ and $R_{E2}$ are calculated from final time forecast vorticity, $\zeta_f$. $E$ is the environmental projection operator. The operator that inverts vorticity and calculates the horizontal wind attributed to that vorticity is $Q^{-1}$. The horizontal wind field attributed to vorticity outside of the response function box is $\tilde{u}_f, \tilde{v}_f$. (b) Flow chart describing how the initial conditions to the adjoint model, $\partial R_{E1}/\partial \zeta_f$ and $\partial R_{E2}/\partial \zeta_f$, are calculated from $\partial R_{E1}/\partial \tilde{u}_f$ and $\partial R_{E2}/\partial \tilde{v}_f$. Operators $E^*$ and $(Q^{-1})^*$ are the adjoints of $E$ and $Q^{-1}$, respectively. Note that for the zonal environmental wind response function, $\partial R_{E1}/\partial \tilde{u}_f = 1$ in the response function box, zero elsewhere, and $\partial R_{E1}/\partial \tilde{v}_f = 0$; while for the meridional environmental wind response function, $\partial R_{E2}/\partial \tilde{v}_f = 1$ in the response function box, zero elsewhere, and $\partial R_{E2}/\partial \tilde{u}_f = 0$. 
Figure 4. (a) Basic state vorticity contoured every $2.0 \times 10^{-5}$ s$^{-1}$ and difference between $\partial R_{E1}/\partial \zeta$ and $\partial R_{1}/\partial \zeta$ shaded every $5.0 \times 10^{4}$ m at model initialization. (b) Basic state vorticity (contoured) and perturbation vorticity shaded every $5.0 \times 10^{-7}$ s$^{-1}$ at model initialization. (c) Perturbation vorticity (shaded) and perturbation winds at model verification. (d) Perturbation environmental vorticity (shaded) and perturbation environmental winds at model verification. The box at the center of the plot corresponds to the boundaries of the response function box.
Figure Captions

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