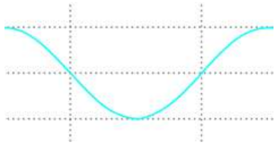
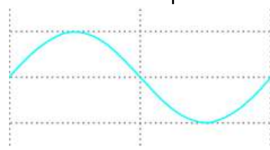
time t=0



time t=pi/2

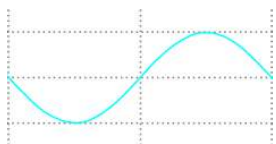


u' -> --> ----> ------> u' <------<----<--<-

v' O O o ∘ v' ∘ o O O

time t=pi

time t=3pi/2



u' <-<--<------ u' ---->-->->

v' x X X X v' X X X x

Each frame above shows a snapshot in time of a wave in a shallow water system. At time t=0, we see the u' component of the vector is at maximum magnitude & pointed to the east while the v' component is 0. Apply this visually in the xy plane and it looks like this:

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At time t=pi/2 seconds later, the frame shows the u' component at 0, and the v' component is at maximum magnitude and pointed to the south. The 2 vectors at times t=0 & t=pi/2 shown on the same graph looks like this:

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|
↓

Tracing all of the vectors from t=0 to t=pi/2 shows an ellipse, with the major axis = $2\omega/$ kH, and the minor axis = 2f/kH. This hodograph traces a parcel's path in the xy plane as a wave passes. This action is important because it shows how a wave in a shallow water system evolves in time. As of yet, we have only described waves at a snapshot in time.

But why must these waves be sinusoidal in nature? This is an assumption we made in the derivation of u' and v' from differential equations. Thanks to Fourier, this assumption covers any function that may evolve because any function can be described in sines and cosines.

$$f(x) = C + \sum_{k=1}^{\infty} a\cos(kx) + \sum_{k=1}^{\infty} b\sin(kx)$$

$$C = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = f$$

$$a = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx$$

$$b = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)dx$$