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AOS 311 Lecture Summary 3/6/09

The concept of invertibility of the quasi-geostrophic potential vorticity equation (QGPV) was introduced. The QGPV (Lagrangian tendency) equation,

$$\frac{D_g}{Dt} q_p = 0$$

which expresses the conservation of QGPV following adiabatic, geostrophic motion is an alternate form of the geopotential tendency equation (equation 6.23 in Holton) introduced earlier.

The form of QGPV used in invertibility is below:

$$q_p' = q_p - f = \frac{1}{f_0}\nabla^2\Phi' + f_0 \frac{\partial}{\partial p}\left(\frac{1}{\sigma}\frac{\partial \Phi'}{\partial p}\right) = \text{"}\nabla_3^2\Phi'\text{"}$$

A **B** **C** **D**

In the above equation, q_p' is the QGPV anomaly, while the primes on the right hand side of the equation denote deviations of geopotential from its horizontal average. The quotes on term D show that this is not a strict equality, but when stratification is assumed to be constant, term C acts like a three dimensional Laplacian of the geopotential height. As a consequence, given q_p and boundary conditions, one can solve for $\vec{V}_g$ and θ' associated with the given q_p. This is very powerful connection, but its usefulness is limited to the mid-troposphere where thickness tendencies near boundaries are small.

Take Home Message: If one knows the PV distribution in the mid-troposphere, one can use invertibility to determine the geostropic winds and temperature.