Key Points from the AOS 311 Lecture of 3/4/09
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The quasi-geostrophic (QG) geopotential tendency equation – derived by combining the QG vorticity equation with the QG thermodynamic equation yields the QG height tendency equation (Holton, p. 155-157). Adiabatic flow (J=0) has been assumed.

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial \phi'}{\partial t} = -f_0 \cdot \nabla\left(f + \zeta_g\right) - \frac{\partial}{\partial p}\left[\frac{-f_0^2}{\sigma}\vec{V}_g \bullet \nabla\left(\frac{\partial \phi}{\partial p}\right)\right],$$

In other words, "the local, three-dimensional geopotential tendency $\left(\nabla_3^2 \frac{\partial \phi'}{\partial t}\right)$ = geostrophic advection of geostrophic vorticity + "differential thickness advection."

The time derivative of geostrophic vorticity and that of the Laplacian of geopotential tendency are related by this equation: $\frac{\partial \zeta_g}{\partial t} = \frac{1}{f_0}\nabla^2 \frac{\partial \phi'}{\partial t}$ If vorticity is increasing locally, then geopotential heights are locally falling.

The quasi-geostrophic vorticity was defined as…

$$q \equiv f + \frac{1}{f_0}\nabla^2 \phi' + f_0 \frac{\partial}{\partial p}\left[\frac{1}{\sigma}\frac{\partial \phi'}{\partial p}\right]$$

$q' \equiv q - f \;\therefore\; q' = \nabla_3^2 \phi'$, where q' is the potential vorticity (PV)

This relationship comes with a few implications discussed in class:

1. If both q' and the boundary conditions are known, the geopotential height distribution can be found.
2. If $q' > 0$, the geopotential height field is at a local minimum.
3. If $q' < 0$, the geopotential height field is at a local maximum.

For example, if there is a PV maximum ($q' > 0$) at the 500-hPa level, the geopotential heights are at a minimum. The temperature is locally colder between the shortened thickness between 500 hPa and the surface. The stretched column of air between 500 hPa and the tropopause is locally warmer. Moreover, the geostrophic wind flows with lower geopotential heights on the left, which in this case, is the center. Thus, cyclonic rotation is induced around the potential vorticity anomaly. In the vertical, deviations from the average geopotential height (ϕ') are at a minimum surrounding the PV anomaly. The winds are maximized on the same height level as the anomaly and decrease with increasing vertical distance away from it.