Lecture Summary for 2 March 2009
AOS 311
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The goal for Monday's lecture was to derive, from the quasi-geostrophic (QG) thermodynamic and vorticity equations, the QG ω-equation – an equation for diagnosing vertical motion.

Deriving the equation

To begin, the Laplacian of the thermodynamic equation (I) was added to the vertical derivative (with respect to pressure) of the quasi-geostrophic vorticity equation (II) :

$$\text{(I)} \quad \frac{1}{f_0}\nabla^2\left[\frac{D_g}{Dt}\left(-\frac{1}{f_0}\frac{\partial \Phi'}{\partial p}\right) = \sigma\omega + \frac{\kappa J}{p}\right]$$

and

$$\text{(II)} \quad \frac{\partial \zeta_g}{\partial t} = -\vec{V}_g \cdot \nabla \zeta_g + f_0 \frac{\partial \omega}{\partial p}$$

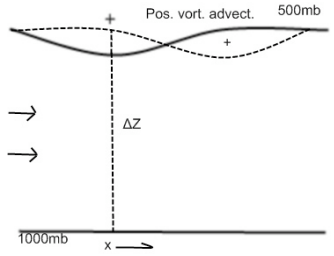
Then separating geopotential, Φ, into vertical and horizontal components and combing the two equations while keeping the ω's, gives the quasi-geostrophic omega equation:

$$\sigma\left(\nabla^2 + \frac{f_0}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = f_0 \frac{\partial}{\partial p}\left[\vec{V}_g \cdot \nabla\left(\zeta_g + f\right)\right] + \nabla^2\left[\vec{V}_g \cdot \nabla\left(-\frac{\partial \phi}{\partial p}\right)\right]$$

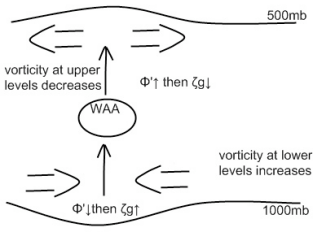
The quasi-geostrophic omega equation is valid for diagnosing ω given an instantaneous geopotential height field. From this equation, a value for ω can be obtained without actually measuring the wind.

When $\nabla^2\omega > 0$ (local minimum)…ω is negative, corresponding to upward vertical motion.
When $\nabla^2\omega < 0$(local maximum)…ω is positive, corresponding to downward vertical motion.

The omega equation illustrated



From the figure to the left, it is clear that positive vorticity advection is associated with increasing vorticity with increasing heights. Since, the thickness of the layer is decreasing, therefore average temperature is decreasing.



In this figure, vorticity advection is turned off and localized warm air advection (WAA) creates a localized increase in thickness).

In essence, vertical and ageostrophic motions are the methods that the atmosphere implements to try to keep itself in hydrostatic and geostrophic balance.