Lecture Summary for 17 April 2009
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With mathematical processes such as scale analysis, complex equations in the atmospheric sciences can be simplified to some degree. Another of these methods is linearization.

With the pendulum example shown in class, it was shown that complex 2nd order differential equations can be simplified with linearization to arrive at approximate solutions.

Linearization of nonlinear variables i.e. $(ab), (aa = a^2)$

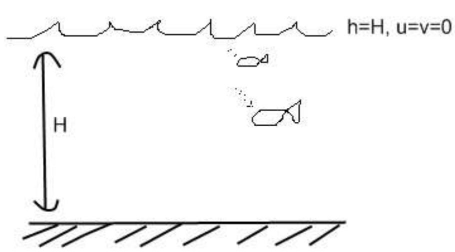
To do a linearization, write each variable as a sum of the basic state solution and the perturbation from that solution.

$$A = \bar{A} + A'$$

Where $\bar{A}$ is the basic state solution, and A' is the perturbation from the solution. Assuming $\bar{A} \gg A'$

Substitute in $A = \bar{A} + A'$ for every variable, then simplify.

Consider the example of linearizing the advective derivative of the horizontal momentum equation from the shallow water system. Note the nonlinear terms. $\left(u\frac{\partial u}{\partial x} \text{ and } v\frac{\partial v}{\partial y}\right)$



$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} - fv = -g\frac{\partial h}{\partial x}$$

Where H=h and $u = v = 0$ at the surface.

Breaking variables into basic and perturbation parts gives… $u = \bar{u} + u',\ v = \bar{v} + v' \text{ and } h = H + h'$

Substituting and canceling basic state terms yields…

$$\frac{\partial u'}{\partial t} + u'\frac{\partial u'}{\partial x} + v'\frac{\partial v'}{\partial y} - fv' = -g\frac{\partial h'}{\partial x}$$

Cancel nonlinear terms results in obtaining the linearized equation of motion for the shallow water system in the horizontal direction.

$$\frac{\partial u'}{\partial t} - fv' = -g\frac{\partial h'}{\partial x}$$

In the future we will look at wave phenomena in the atmosphere as well as linearizing these oscillatory solutions as a function of time—much like the pendulum example.