Stephen Ogden

3/27 Lecture Summary

The goal of this lecture was to find a relation between the average of a value and its perturbation, and to create a closed system of equations. First, we defined average momentum equations, the zonal equation being

$$\frac{\overline{Du}}{Dt} = \frac{-1}{\rho_0}\frac{d\bar{p}}{dx} + f\bar{v} - \left[\frac{d}{dx}(\overline{u'u'}) + \frac{d}{dy}(\overline{u'v'}) + \frac{d}{dz}(\overline{u'w'})\right] + \overline{F_{rx}}$$

and an average thermodynamic equation

$$\frac{\overline{D\theta}}{Dt} = -\bar{w}\frac{d\bar{\theta}}{dz} - [\frac{d}{dx}(\overline{\theta'u'}) + \frac{d}{dy}(\overline{\theta'v'}) + \frac{d}{dz}(\overline{\theta'w'})]$$

Since the variables are $\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta}, u', v', w', p', \theta'$, and not u, v, w, p, θ, we cannot get a closed system without having the values of the prime variables. We can try to get a relation, in a parameterization problem for turbulence, or a closure problem.

We found that the turbulent energy of eddies per unit mass was $\frac{1}{2}[u'^2 + v'^2 + w'^2]$ and that the derivative of it is made up of four parts: buoyancy production/loss, momentum production, transport, and viscous dissipation, which are related by

$$\frac{\bar{D}}{Dt}TKE = \overrightarrow{u'}\left(\frac{D\vec{u}}{Dt} - \frac{\overline{D\vec{u}}}{Dt}\right) = BPL + MP + TR - E$$

Transport is not related to the generation of energy. Viscous dissipation is always positive, as it acts on eddies using friction to lessen their kinetic energy. Buoyancy production/loss is $\frac{g}{\theta}(\overline{w'\theta'})$.
When air is able to convect, eddies form by air rising into an area where it is warmer than the surrounding air or sinking into an area where it is colder than the surrounding air. It then causes a convective roll due to conservation of mass. Momentum production is created by shear. This term is $-\overline{u'w'}\frac{d\bar{u}}{dz} - v'w'\frac{d\bar{v}}{dz}$.
If w'>0 and u'<0, or w'<0 and u'>0, then $\overline{u'w'} < 0$, and there is eddy growth due to the movement of stronger winds from higher heights down to the surface and movement of calm winds from the surface to higher heights. If $\overline{u'w'} < 0$ when $\frac{d\bar{u}}{dz} > 0$, then $\overline{u'w'} = -k\frac{d\bar{u}}{dz}$, and $\frac{d}{dz}(\overline{u'w'})$ is the only important term in the momentum equation. This means $\frac{d}{dz}(\overline{u'w'}) = \frac{d}{dz}(-k\frac{d\bar{u}}{dz})$, and the momentum equation becomes $\frac{\overline{Du}}{Dt} = \frac{-1}{\rho_0}\frac{d\bar{p}}{dx} + f\bar{v} - k\frac{d^2\bar{u}}{dz^2} + \overline{F_{rx}}$.