A global barotropic spectral model

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AOS 471: Numerical Techniques in Weather Prediction

- **Course description:** Introduction to mathematical aspects of numerical weather prediction (NWP) models. Programming done using Matlab and FORTRAN 77.

There are six principle course objectives:
- To understand the fundamental components (observations, assimilation system, modeling system, and post-processing systems) of an NWP system.
- To gain experience in developing and applying finite difference and spectral methods to the solutions of simple NWP-related problems.
- To develop the skills necessary to set-up, run, and diagnose the output of a mesoscale NWP model.
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- To understand the formulation, solution, and interpretation of statistical analyses of model output.
- To understand the limitations of (operational) NWP models
- To understand how model output may be evaluated for skillfulness.
Differences between FD and spectral models

Consider the linear advection equation:

\[ \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \]
with I.C. \( u(x,0) = u_0(x) \)

Finite difference approach involves a discretization of the forecast variable and equation in space and time with:

\[ u(x,t) = u(x_j,t_n) = u^n_j \]

The linear advection equation may then be written:

\[ u^{n+1}_j = u^{n-1}_j - c \frac{\Delta t}{\Delta x} (u^n_{j+1} - u^n_{j-1}) \]
Alternatively, we may write

\[ u(x,t) = u(x_j,t) = \sum_{k=0}^{N} c_k \hat{u}_j(t) e^{ikx_j} \]

where \( \hat{u}_j(t) \) are the discrete Fourier transform expansion coefficients of \( u(x,t) \).

The linear advection equation may then be written:

\[ \frac{d\hat{u}_k}{dt} + cik\hat{u}_k = 0 \]

or, after discretizing in time:

\[ \hat{u}_k^{n+1} = \hat{u}_k^{n-1} - 2\Delta t(ick\hat{u}_k^n) \]
Barotropic Vorticity Equation

The two-dimensional, non-divergent barotropic vorticity equation (BVE) states that absolute vorticity, $\eta$, is conserved following the 2D non-divergent flow:

$$\frac{D\eta}{Dt} = 0$$

Written in terms of the streamfunction, the Eulerian form of this equation is:

$$\frac{\partial \nabla^2 \psi}{\partial t} = \frac{1}{a^2 \cos \phi} \left( \frac{\partial \psi}{\partial \phi} \frac{\partial \nabla^2 \psi}{\partial \lambda} - \frac{\partial \psi}{\partial \lambda} \frac{\partial \nabla^2 \psi}{\partial \phi} \right) - \frac{2 \Omega}{a^2} \frac{\partial \psi}{\partial \lambda} = F(\lambda, \mu)$$

Expressing the streamfunction and advection in terms of (triangularly truncated) spherical harmonics:

$$\psi = \sum_{m=0}^{N} \sum_{n=|m|}^{N} \hat{\psi}_n^m P_n^m(\mu) e^{im\lambda}$$

$$F = \sum_{m=0}^{N} \sum_{n=|m|}^{N} \hat{F}_n^m P_n^m(\mu) e^{im\lambda}$$
results in:

\[ - \frac{n(n+1)}{a^2} \frac{d\hat{\psi}_m^n}{dt} = \hat{F}_m^n \]

Discretizing in time (Leap Frog):

\[ (\psi_m^n)^{\tau+1} = (\psi_m^n)^{\tau-1} - \frac{2a^2}{n(n+1)} \Delta t (\hat{F}_m^n)^\tau \]

**Issues:**
1. What are spherical harmonics?
2. Understanding the Fourier-Legendre transform and related spectral truncation
3. How are non-linear advection terms computed?
Fourier-Legendre transform

\[ A(\lambda_j, \phi_k, t) = \sum_m \sum_n \hat{A}_n^m(t) P_n^m(\mu_k) e^{im\lambda_j} = \sum_m \sum_n \hat{A}_n^m(t) Y_n^m(\lambda_j, \mu_k) \]

Step 1: Discrete Fourier transform \( A \) along latitude circles

\[ A_m(\mu_k, t) = \frac{1}{2M} \sum_{j=0}^{2M+1} A(\lambda_j, \mu_k, t) e^{-im\lambda_j} \text{ where } \lambda_j = \frac{\pi}{M} j \]

Step 2: Legendre transform \( \hat{A}_m \) in the north-south direction

\[ \hat{A}_n^m(t) = \sum_{k=1}^K w(\mu_k) A_m(\mu_k, t) P_n^m(\mu_k) \]

Above image from http://www.du.edu/~jcalvert/math/harmonic/harmonic.htm
Triangular truncation

For any spectral representation of a field, \( A(\lambda, \phi, t) \), \( n \leq m \). ‘m’ is the zonal wavenumber, ‘n’ is the degree of the associated Legendre polynomial, ‘n-m’ is like a meridional wavenumber.

\[
A(\lambda, \phi, t) = \sum_{m=0}^{N} \sum_{n=|m|}^{N} \hat{A}_n^m(t) P_n^m(\mu) e^{im\lambda}
\]
Set up

• Create your own subdirectory: `mkdir yourlastname`

• `cd` into that directory `cd yourlastname`

• Untar the code and related scripts `tar --xvf /data/morgan/morgan_bve.tar`

• `cd` into the spectral subdirectory `cd spectral`

• Edit `*.sh` files to define the current directory

Code will be also available from http://aurora.aos.wisc.edu/~morgan/bsm.html
Part 1. Spectral truncation

Part 1. *Spectral decomposition (using local data)*

- *make all*
- Run `./fnl_tri_trunc.sh 060209 00 80` (will create a T80 truncation of the NCEP final analysis of relative vorticity and place results in a file t80.gem)
- Run *gdplot*; restore settings from the file trunc (i.e., *restore trunc*)
- Experiment with different resolutions – note the vorticity and streamfunction structure. *Note that each time you run fnl_tri_trunc.sh this script, a new GEMPAK file tmmm.gem will be created; as a consequence, in gdplot, you’ll have to change gdfile.*
The model

Initial vorticity from GFS → Initial streamfunction

Transform streamfunction

Calculate forcing on Gaussian grid

Time step ahead for new streamfunction

Output at 3 hour intervals
Part 2. Running the model

Part 2. Barotropic spectral model (using local data)

• Run .!/fnl_bve_t108.sh 060209 00 (this will compile the BVE model and have it run out for 48 hours starting from 0000 UTC 9 February 2006). The model output is then converted into a GEMPAK grid file (psi.gem).

• cd to the model directory (cd model).

• The program gdplot may be used to view psi.gem (restore bve).

• Try to rerun the model with the data from 1200 UTC 29 August 2005.
Part 3. Running the model in near real-time

- Run "./bve_t108.sh 060209 00" (this will compile the BVE model and have it run out for 48 hours starting from 0000 UTC 9 February 2006). The model output is then converted into a GEMPAK grid file (psi.gem).
- `cd` to the model directory (`cd model`).
- The program `gdplot` may be used to view `psi.gem` (restore bve).
- `cd ..` (to go back up a subdirectory level)
- Run `gdplot` (restore gfs)
Possible exploration

• Look at 12 hr forecast error – run
  
  ./bve_48h_t108.sh 060712 06

• cd model, run gdplot after (restore error)

• How might you explain these forecast differences? What information do you need?
Possible next steps with model

- Add divergence/convergence forcing associated with stretching over topography
- Add numerical diffusion to get rid of small scale “noise”