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Last lecture we began to move into a new part of the syllabus, where we began to introduce some material that will be covered in greater detail in AOS 452.

The main theme of the lecture was beginning to develop a set of quasi-geostrophic equations. This is done by scaling of the governing equations to *O(R$_o$)*. Our ultimate goal will be to derive an equation for vertical motion – the quasi-geostrophic -- ω -- equation.

First, we must assume that the flow is geostrophic. We define a *constant f* geostrophic wind as ($\mathbf{V_g} = \frac{1}{f_o}\mathbf{k}\times\nabla\Phi$). Then we assume that $\frac{|\mathbf{V_{ag}}|}{|\mathbf{V_g}|} \cong O(R_o)$, which is equivalent to assuming $|\mathbf{V_{ag}}| << |\mathbf{V_g}|$ for small Rossby number. Then, since $\mathbf{V} \cong \mathbf{V_g}$, we can replace $\mathbf{V}$ with $\mathbf{V_g}$ in the momentum equation, giving us the *geostrophic momentum approximation*:

$$\frac{D}{Dt}\mathbf{V}_g + f\mathbf{k}\times\mathbf{V} = -\nabla\Phi$$

We can also make use of the the β -plane approximation for which we assume that the Coriolis parameter varies linearly with latitude: $f = f_o + \beta_o y_o$

This approximation is valid provided that $\frac{\beta_o L_y}{f_o} << O(Ro)$ where L_y is the characteristic meridional length-scale for weather or ocean disturbances we are considering.

The β - plane approximation is then valid if $\dfrac{\frac{2\Omega\cos\phi}{a}L_y}{2\Omega\sin\phi}$ The 2Ω terms cancel out, which leaves us with $\frac{L_y}{a}$, which scales like the Rossby number as long as L_y is much smaller than the radius of the earth. With these assumptions made, the quasi-geostrophic equation is then:

$$\frac{D_g}{Dt}\mathbf{V_g} = -f_o\mathbf{k}\times\mathbf{V_{ag}} - \beta_o y\mathbf{k}\times\mathbf{V_g}$$