

This lecture continued to focus on linearized equations and the propagation of different wave structures, such as gravity waves. We began with the equations for \hat{u} and \hat{v} where

$$\hat{u} = \frac{g\hat{h}}{\omega^2 - f^2}(\omega k - if\ell), \quad \hat{v} = \frac{g\hat{h}}{\omega^2 - f^2}(-ifk + \omega\ell),$$

and we assume that \hat{h} is real. Then \hat{h} is equal to the real part of the equation $h' = \hat{h}e^{i(kx + \ell y - \omega t)}$ which equals $\hat{h}\cos(kx + \ell y - \omega t)$. If k and ℓ are positive and $t=0$, then $h' = \hat{h}\cos(kx + \ell y)$. This setup exhibits wavelike behavior in both the horizontal as well as the meridional directions.

If $\ell = 0$ and $\hat{h} > 0$ and is real, then $h' = \hat{h}\cos(kx - \omega t)$ and $u' = \frac{g\omega k}{\omega^2 - f^2}\cos(kx - \omega t)$. Through

a series of substitutions, we come to the solution that $u' = \frac{w\hat{h}}{Hk}\cos(kx - \omega t)$. Using these same

conditions we can also see that $v' = \frac{f\hat{h}}{Hk}\sin(kx - \omega t)$. These equations, in this situation, produce

gravity waves. These are maintained by divergence and propagate in the direction of u' . We can show

that these waves are related to divergence in the following way. We know that $\zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}$, which

in turn equals $\frac{f\hat{h}}{H}\cos(\dots)$. Then, $\delta' = \frac{\partial u'}{\partial x} - \frac{\partial v'}{\partial y}$, which equals $\frac{\hat{h}\omega}{H}\sin(\dots)$. We can then take $\frac{|\zeta'|}{|\delta'|} = \frac{f}{\omega}$

When this value is less than one, then that means that $\omega \gg f$, which means that this case is valid when the vertical motion is much greater than the Coriolis parameter. When the vertical motion is high, then there must be divergence somewhere because the air cannot move vertically forever. This divergence is what then propagates the motion of these waves.