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AOS 311

Feb. 23 Lecture Summary

We began by looking at Rossby's proposed solution to the two-dimensional, non-divergent, barotropic vorticity equation:

$$\psi = -Uy + A\sin(k(x - ct))$$

where the first term on the right-hand side represents the streamfunction associated with a zonal flow and the second term represents a wave solution. We were reminded that the wavelength is inversely proportional to the wavenumber: $L_x = 2\pi/k$.

To understand what happens with the Rossby wave when there is no zonal component to the flow, we assume that there is no initial relative vorticity so that $\zeta_2 = -\beta\delta y$ which shows that the vorticity depends on variations in latitude. From this equation we can see that if $\delta y > 0$ (northward perturbation), then $\zeta_2 < 0$ which corresponds to anticyclonic vorticity and if $\delta y < 0$ (southward perturbation), then $\zeta_2 > 0$ which corresponds to cyclonic vorticity. Furthermore, the greater the magnitude of δy, the greater the magnitude of ζ_2.

We then investigated what happens with the Rossby wave under two distinct situations; one where *k* is large, corresponding to a small scale feature, and one where *k* is small, corresponding to a large scale feature. With $\zeta = \nabla^2\psi$, we got an equation for the streamfunction that varied depending upon *k*

$$\psi_k = -\frac{\hat{\zeta}_k}{k^2} * \sin kx$$

where $\hat{\zeta}_k$ is the amplitude of the vorticity wave. If we take the derivative, we obtain an equation for the velocity

$$V_k = -\frac{\hat{\zeta}_k}{k} * \cos kx$$

These equations also show that the maximum winds are always 90 degrees down the wave from the maximum vorticities because of the sine-cosine relationship (strongest winds at inflection points of waves while strongest vorticities at the peaks). Expanding the Eulerian time derivative of the vorticity and neglecting the zonal component of the flow, we arrive at

$$\frac{\partial \zeta}{\partial t} = -v\beta = \frac{\hat{\zeta} * \cos kx}{k} * \beta$$

When we bring *U* back into the equation, the time rate of change of vorticity becomes

$$\frac{\partial \zeta}{\partial t} = -U\frac{\partial \zeta}{\partial x} - v\beta = -U\hat{\zeta}k * \cos kx + \frac{\hat{\zeta} * \cos kx}{k} * \beta$$

and we found that small scale perturbations move westward less rapidly than large scale perturbations because in the equation relating the phase speed and the beta effect: $c = U - \frac{\beta}{k^2}$, features with small wavelengths (L_x is small or *k* is large) then the β/k^2 term will be smaller which will make *c* less negative, meaning less westward movement, which corresponds to greater eastward movement. We also saw evidence of this when we looked at the forecast models because small scale features moved much faster than large scale features and the large features appeared to be nearly stationary as well.