

Kristin Klingelhutz
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Summary of Lecture from Friday, 20 February

Rossby Waves

The two-dimensional, barotropic vorticity equation (BVE) expresses conservation of absolute vorticity $\frac{D}{Dt}(f + \zeta) = 0$.¹ On a β -plane the Eulerian time rate of change of ζ can be expressed as

$$\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial x} - v \frac{\partial \zeta}{\partial y} - v \beta$$

where the first^t and second terms describe the advection of relative vorticity by the horizontal wind, and the third term is the advection of planetary vorticity by the meridional wind.

Again, for a 2D, non-divergent flow, one can define a streamfunction, ψ , such that $\zeta = \nabla^2 \psi$ so that the Eulerian form of the BVE becomes:

$$\frac{\partial \nabla^2 \psi}{\partial t} = \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} \nabla^2 \psi - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \nabla^2 \psi - \frac{\partial \psi}{\partial x} \beta$$

Rossby proposed the solution $\psi = -Uy + A \sin[k(x - ct)]$ to the BVE. In this solution, U represents the speed of a (background) current, while A , k , and c represent the amplitude, zonal wave number, and zonal phase speed of a wave embedded in this current. The

phase of the wave is $kx - ct$. The zonal phase speed of the wave $\frac{dx}{dt} = c$ is the speed you

have to move in order to keep the phase of the wave constant. If $c > 0$ then the wave moves to the east and if $c < 0$ the wave moves west. Substituting the proposed solution into the BVE we obtain a relationship between the zonal phase speed and zonal

wavenumber, which is: $c = U - \frac{\beta}{k^2}$. We note that the amplitude of the wave is not

relevant. Eastward propagation occurs for $c > 0$, that is for $k > \sqrt{\frac{\beta}{U}}$ (i.e., small-scale

waves). The last term, $\frac{-\beta}{k^2}$, is known as the beta effect.

¹ The vorticity for northward displacement is negative (clockwise) and the vorticity for a southward displacement is positive (counter-clockwise).

The fluid parcels oscillate back and forth about their equilibrium latitude, and the pattern of vorticity maxima and minima propagates to the west. This propagation constitutes the Rossby wave.