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Lecture Summary from 4/13/09

Key Concept: Surface Ekman Layer Problem

The surface layer, whose depth depends on stability, but is usually less than 10% of the total boundary layer depth, is maintained entirely by vertical momentum transfer by the turbulent eddies; it is not directly dependent on the Coriolis or pressure gradient forces (Holton, 129).
Along the ocean surface there is a horizontal frictional stress from wind.
Assuming steady conditions, a homogeneous fluid, and a geostrophic interior you can assume the following equations and boundary conditions for the flow field in the surface Ekman layer:

$$-f(v - v_g) = K\frac{\partial^2 u}{\partial z^2} \qquad\qquad -f(u - u_g) = K\frac{\partial^2 v}{\partial^2 z}$$

Boundary Conditions:

surface (z=0) and toward interior z=-∞ u->u_g and v-> v_g

Wind Stress:

$$\rho_0 k\frac{\partial u}{\partial z} = \tau^x \text{ the zonal wind stress} \quad \rho_0 k\frac{\partial v}{\partial z} = \tau^y \text{ the meridional wind stress}$$

Solving for u and v which is the total current gives

$$u = u_g + \frac{\sqrt{2}}{\rho_0 f d}e^{z/d}(\tau^x \cos(\frac{z}{d} - \frac{\pi}{4}) - \tau^y \sin(\frac{z}{d} - \frac{\pi}{4}))$$

$$v = v_g + \frac{\sqrt{2}}{\rho_0 f d}e^{z/d}(\tau^x \cos(\frac{z}{d} - \frac{\pi}{4}) + \tau^y \sin(\frac{z}{d} - \frac{\pi}{4}))$$

where d is $d = \sqrt{\frac{2k}{f}}$

where everything to the right of geostrophic wind is the wind driven part of the solution.

The wind-driven flow component is inversely proportional to the Ekman-layer depth (d). When d is small (thus small eddy viscosity) the wind driven part of the solution gets very large, which can generate large velocities.
The wind driven horizontal transport in the surface Ekman layer has components given by:

$$U = \int_{-\infty}^{0}(u - u_g)dz = \frac{1}{\rho_0 f}\tau^y \quad \text{and} \quad V = \int_{-\infty}^{0}(v - v_g)dz = -\frac{1}{\rho_0 f}\tau^x$$

Surprisingly, the transport is oriented perpendicular to the direction of the wind stress. Just to clarify, the wind stress (τ^x and τ^y) blow in the same direction as the surface wind, the net transport is what is 90 degrees to the right in the Northern hemisphere and 90 degrees to the left in the Southern hemisphere.

If the wind stress has a nonzero curl, the divergence of the Ekman transport must be provided by a vertical velocity throughout the interior. To find this, vertically integrate the continuity equation across the Ekman layer with w(z=0) and w(z->-∞). A + curl of the wind stress=upwelling and a – curl of the wind stress = down welling.