Greg Kipnis Feb 20th Lecture Summary

MATLAB

We began class with a quick tutorial for plotting functions in Matlab. To assign variables you can set your variable equal to a sequence of values with an upper and lower bound for your data along with increments all separated by colon signs.
typing a semi colon after a formula will suppress the result from showing up in the Matlab workspace
typing "plot" and then the variables will display all the data on a two dimensional plot
typing "clear all" will erase all variables that have been created
typing "whos" displays all variables that you have defined

Time Rate of Change of Vorticity

We performed a scale analysis for the time rate of change for the vorticity equation and were able to eliminate the zeta term demonstrating that that the Lagrangian time rate of change of the vertical component of absolute vorticity is equal to $-f$ multiplied by the divergence

$$\frac{D_h}{Dt}(f + \zeta) = -f\delta$$

A 2D scale analysis tells us that the divergence is small so we assume that the flow is effectively non-divergent,

$$\frac{D_h}{Dt}(f + \zeta) = 0$$

This is the two-dimensional, non-divergent barotropic vorticity equation (BVE). Major Assumptions: Flow is 2D and non divergent, thus divergence=0 meaning absolute vorticity is conserved. This is equivalent to taking the shallow water system and putting lid on it so there is no stretching. We then expanded the BE into its form:

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - v\beta$$

We then want to figure out how vorticity changes at a specific location over time. Because the latitude remains constant f, which is a function of latitude, is constant and can be eliminated from the equation.
For a two-dienstional, nondivergent flow we can define a stream function . In terms of the streamfunction, the vorticity

$$\zeta = \nabla^2\psi$$

We now substitute back into the vorticity equation in terms of the stream function

$$\frac{D_h}{Dt}(f + \nabla^2\psi) = 0$$

Expanding this out into the local Eulerian derivative and setting this result equal to the change in vorticity over a period of time allows us to predict how vorticity will change over time.

$$\frac{\partial \nabla^2\psi}{\partial t} = \frac{\partial\ \psi}{\partial y}\frac{\partial \nabla^2\psi}{\partial x} - \frac{\partial\ \psi}{\partial x}\frac{\partial \nabla^2\psi}{\partial y} - \frac{\partial\ \psi}{\partial x}\beta \cong \frac{\zeta_{new} - \zeta_{old}}{\Delta t}$$

In principal, given a value of vorticity at time $t = 0$, we can predict how vorticity will change with time in an unbounded domain, or in a closed domain with specified boundary conditions. The BE formed the basis for the first numerical weather prediction models.
The lecture ended with a n introduction to Rossby's proposed solution to the BE

$$\psi = -Uy + A\sin k(x - ct)$$

where U, A, k, and c are all constants representing the magnitude of a zonal flow, amplitude of a wave component of the flow, zonal wave number #, and the zonal phase speed, respectively.