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Lecture 3/30

The goal of this lecture was to come up with a mixed layer solution for understanding the behavior of winds within a well mixed boundary layer.

The Flux Richardson number is a measure of dynamic stability within the atmosphere. This number is represented by the ratio of the Buoyancy Production Lost and Momentum Production lost terms,

$$R_{if} = -\frac{BPL}{MP}$$

For values of $R_{if} < 0$ turbulence will be convectively generated. For values $0 < R_{if} < \frac{1}{4}$, turbulence will be generated by momentum production lost. For all values $R_{if} > \frac{1}{4}$ then no turbulence will be present. We then looked to find a relationship between the prime'ed variables and bar'ed variables looking at the momentum equation.

$$\frac{D\overline{u}}{Dt} = \frac{1}{\rho_\circ}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \left[\frac{\partial}{\partial x}\left(\overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\overline{u'w'}\right) + \overline{F_{rx}}\right]$$

Assuming that the turbulence is horizontally homogeneous, we can determine that the convergence of turbulent eddy fluxes in the horizontal directions will be smaller than the vertical direction. Thus, these terms go to zero as seen by a simple scale analysis in which the larger horizontal terms drop out exposing the vertical term as the dominant one. Additionally, we can eliminate the F_{rx} term since frictional forces can be neglected because we are looking at a flow above the viscous sublayer. Lastly, $\frac{D\overline{u}}{Dt}$ can be eliminated assuming that $R_{\circ T}, R_\circ << 0$ relative to the other forces present. Because this is a boussinesq fluid, the geostrophic wind is independent with respect to height allowing a substitution of v_g for the PGF giving us two planetary boundary layer equations,

$$\frac{\partial}{\partial z}\left(\overline{u'w'}\right) = f(\overline{v} - \overline{v}_g) \text{ and } \frac{\partial}{\partial z}\left(\overline{v'w'}\right) = -f(\overline{u} - \overline{u}_g)$$

Given $\overline{u_g}, \overline{v_g}$ at the surface we want to find $\overline{u}, \overline{v}$ as a function of height assuming $f = f_\circ$ and $\rho = \rho_\circ$ in order to determine the wind profile in the higher atmosphere. We then defined the bulk aerodynamic drag formula which set the average transport of momentum along the surface equal to the wind speed at an anemometer level along with accounting for a drag coefficient,

$$\left(\overline{u'w'}\right)_{sfc} = C_d\left|\overline{\vec{v}}\right|\overline{u}$$

Integrating the two equations above from the surface up to the top of the boundary layer we get

$$-C_d\left|\overline{\vec{v}}\right|\overline{u} = f(\overline{v} - \overline{v}_g)$$

This gives us one mixed layer solution which explains what winds look like within a mixed layer,

$$\overline{u} = \overline{u}_g - K\left|\overline{\vec{v}}\right|\overline{v} \text{ and } \overline{v} = K\left|\overline{\vec{v}}\right| \text{ where } K_d = \frac{C_d}{f\,h}$$

The flow within this mixed layer moves in a direction nearly parallel to isobars with a component moving toward lower pressure which is dependent on K. The higher the drag coefficient the more the flow deviates from its intended path along the isobars.