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Lecture Summary for May 1st, 2009

The key emphasis for this lecture was the effects arising from adding in the effects of the - plane into our linearized shallow water system case. The sinusoidal motions of importance from this are Rossby Waves, which were discussed earlier this semester. The addition of the -e ffects into our shallow water equations provides the following momentum equations for this system:

$$\frac{\partial u'}{\partial t} - (f_0 + \beta_0 y) * v = -g\frac{\partial h'}{\partial x}$$

$$\frac{\partial v'}{\partial t} + (f_0 + \beta_0 y) * u = -g\frac{\partial h'}{\partial y}$$

$$\frac{\partial h'}{\partial t} + H(u_x + v_y) = 0$$

Assuming that frequency is way less than the planetary vorticity, or Coriolis parameter ($_0$<<f_0), R_{OT}~O(1), and that the length scale parameter L << 1 such that (*L /f_0) << 1, the following horizontal momentum equations are derived:

$$v' \cong \frac{g}{f_0} * \frac{\partial h'}{\partial x} - \frac{g}{f_0^2}\frac{\partial^2 h'}{\partial y \partial t} - \frac{\beta_0 g y}{f_0^2}\frac{\partial h'}{\partial x}$$

$$u' \cong \frac{-g}{f_0} * \frac{\partial h'}{\partial y} - \frac{g}{f_0^2}\frac{\partial^2 h'}{\partial x \partial t} + \frac{\beta_0 g y}{f_0^2}\frac{\partial h'}{\partial y}$$

where the first terms on the RHS of the equations are the geostrophic balance terms, the second terms on the RHS are the isallobaric wind terms, and the final set of terms on the RHS is the -c orrection effect terms for u' and v' respectively. Recall that the isallobaric wind is defined as the wind velocity that is in balance between the Coriolis force and an accelerating geostrophic wind, traveling normal to the isobars from an area of high to low tendency (divergence → convergence).

Using the mass continuity equation of this system, we find a general solution for the frequency our linearized shallow-water system with - plane effects:

$$\omega = \frac{-\beta_0 k}{\left(\frac{1}{L_r^2}\right) + (k^2 + l^2)}$$

with k and l equal to the zonal and meridional wavenumbers, respectively, and L_r equal to the distance a gravity wave travels in one day. From this along with the scale analysis of , we see that

$\omega \sim -\beta_0 L \ll f$ (for L << L_r) and $\omega \sim -\beta_0 \left(\frac{L_r}{L}\right) L_r \ll f$ (for L >> L_r)

This supports the assumption that < <f, and due to this, confirms that Rossby Waves are a "slow" wave motion that is nearly in geostrophic balance, and that these occur over large timescales, feeling the effects of the Earth's rotation. Although gravity waves still exist in this situation, they have been filtered out for simplicity.

Key Points:

- A linearized shallow-water system including -pl ane effects is most influenced by the Coriolis force and pressure gradient force, with the horizontal perturbation wind defined by geostrophic, isallobaric and - correction components (second and third terms on RHS are ageostrophic wind components).
- The isallobaric wind flows from areas of high to low tendency (from areas of divergence to convergence), increasing in magnitude based on the gradient of the tendency provided.
- Rossby Waves are slow moving waves that are nearly geostrophic and on a large timescale, with < <f.
- $\omega \propto \frac{1}{(wavenumber)}$; smaller waves lead to higher frequencies, and vice versa.