

Lecture summary Friday 2/13/09

f-plane approximation: $f = 2\Omega \sin \phi = f_0$

Assume that we are treating the Coriolis parameter as a locally constant value. Where f_0 is the first term in a Taylor series expansion of f . It is a reasonable assumption for local phenomena, where variation in f is very small. However, if we are analyzing weather over a large area, this f -plane assumption is not sufficient. It doesn't take into account variation of f in the Northward or Southward directions.

 β -plane approximation,

Allow f to vary linearly with latitude, in which we take more than just the first term in a Taylor series expansion of $f \Rightarrow \left[f = f_0 + \frac{df}{dy}(y - y_0) + \dots \right]$, where $\beta = \frac{df}{dy}$.

β -plane approximation of f is appropriate when we are looking at large scale fluctuations, in which the weather systems cover thousands of kilometers.

Rossby Waves

Applying conservation of potential vorticity to the situation of zonal, unshered flow crossing an infinite mountain range revealed that a stationary wave that is generated downstream of that mountain range. These waves do not exist on an f -plane approximation scheme, they occur because of the gradient of planetary vorticity, so we will allow f to vary with latitude.

#Note: correction: $\frac{DC_a}{Dt} = -\oint \frac{dp}{\rho} = +\iint \frac{\nabla \rho \times \nabla p}{\rho^2} \cdot d\mathbf{A}$ NO (-) sign in front of the integral!

Vorticity and the vorticity equation

Vorticity is a vector, $\boldsymbol{\omega}$, and is defined as the curl of the velocity field:

$$\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \mathbf{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) \mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}$$

We derived a vorticity equation, an expression for the Lagrangian time rate of change of absolute vorticity ($f + \xi$), by taking the curl of the horizontal momentum equations. We found that the inviscid vorticity equation contained three terms:

1. a convergence/divergence term: $-\left[(f + \xi) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right]$
2. a tilting/twisting term (often associated with genesis of tornadoes): $-\left[\left(\frac{\partial w}{\partial x} \frac{\partial v}{\partial z} \right) - \left(\frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) \right]$
3. a solenoidal term (baroclinic) generation or sink term: $+\left[\frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right) - \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) \right]$