We developed a set of equations for a frictionless, boussinesq, shallow water system at rest to see what would happen if we apply small perturbations. The linearized results were a system of equations for a shallow water system at rest (**No Advection).**

$$\frac{\partial u'}{\partial t} - fv' = -g\frac{\partial h'}{\partial x} \quad , \quad \frac{\partial v'}{\partial t} + f' = -g\frac{\partial h'}{\partial y} \quad , \quad \frac{\partial h'}{\partial t} + H\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0$$

Using the knowledge that constant coefficient differential equations have exponential solutions, and assuming the solutions will exhibit wavelike structure, we can determine u', v', h' to have the form…(Where R_e is a real part of the function).

$$u' = [\hat{u}e^{i(Kx+\ell y=\omega t)}]R_e$$
$$v' = [\hat{v}e^{i(Kx+\ell y=\omega t)}]R_e$$
$$h' = [\hat{h}e^{i(Kx+\ell y=\omega t)}]R_e$$

- Where substituting **(A)** into the above eqn., differentiating and rearranging yields

$$\begin{pmatrix} -i\varpi\hat{u} - f\hat{v} + giK\hat{h} = 0 \\ f\hat{u} - i\omega v' + gi\ell\hat{h} = 0 \\ HiK\hat{u} + Hi\ell v' - iw\hat{h} = 0 \end{pmatrix}$$

- This is a linear algebraic equation. If the determinant of the matrix is zero, there are non-trivial solutions…

$$Det\begin{bmatrix} -i\omega & -f & giK \\ f & -i\omega & gi\ell \\ HiK & Hi\ell & -i\omega \end{bmatrix} = \omega^3 - \omega\left[gH\left(K^2 + \ell^2\right) + f\right] = 0$$

<u>3 solution frequencies:</u>

- $\omega = 0$
  - Steady Solution: $\frac{\partial}{\partial t} \Leftrightarrow -i\omega$
  - Geostrophic Balance, non-divergent
- $\omega = \pm\sqrt{f^2 = gh\left(K^2 + \ell^2\right)} \Rightarrow$ **If** $|\omega| > f$
  - Gravity waves (fast motions)