Lecture Summary Monday 10 Feb 09

The two main topics covered were an application of Kelvin's Circulation Theorem and the introduction of Ertel Potential Vorticity.

The first main point was devoted to deriving Kelvin's Circulation Theorem. We started with introducing the notion of an autobarotropic fluid (a baroclinic fluid that behaves as if it were barotropic) as a particular case of a flow along an isentropic surface (one with constant potential temperature). Using the Poisson's formula for potential temperature, and the equation of state, we showed how density is a function solely of pressure (only on isentropic surfaces, i.e., surfaces of constant potential temperature). In this case, you can show that

$$\oint \frac{dp}{\rho} = C \oint \frac{dp}{p^{C_v/C_p}} = 0$$

We know that the integral equals zero because the right hand side is a closed line integral of an exact differential. So it's conserved. Using what we know about the relationship between Ca, Cr, and the left hand side of the integral, we can rearrange the equation to show

$$0 = \oint \frac{dp}{\rho} = \frac{DC_a}{Dt} = \frac{D}{Dt}(C_r + 2\Omega \sin(\phi) A)$$

This is known as Kelvin's Theorem. It shows that the absolute circulation following a parcel is conserved.

The second portion of lecture, we combined Kelvin's Theorem for flow along an isentropic surface with conservation of mass, to derive Ertel Potential Vorticity. It is written as

$$q \equiv -g \frac{\partial \theta}{\partial p}(f + \zeta_\theta)$$