A global barotropic spectral model

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Differences between FD and spectral models

Consider the linear advection equation:

\[
\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad \text{with I.C.} \quad u(x,0) = u_0(x)
\]

Finite difference approach involves a discretization of the forecast variable and equation in space and time with:

\[
u(x,t) = u(x_j,t_n) = u^n_j
\]

The linear advection equation may then be written:

\[
u_{j}^{n+1} = u_{j}^{n-1} - c \frac{\Delta t}{\Delta x} (u_{j+1}^{n} - u_{j-1}^{n})
\]
Alternatively, we may write

\[
\hat{u}(x,t) = \sum_{j=1}^{N} \left[ \hat{a}_j(t) \cos(nkx) + \hat{b}_j(t) \cos(nkx) \right] = \sum_{j=1}^{N} \hat{u}_j(t)e^{ijkx}
\]

where \( \hat{u}_j \) are the discrete Fourier transform expansion coefficients of \( u(x,t) \).

The linear advection equation may then be written:

\[
\frac{d\hat{u}_j}{dt} + cik\hat{u}_j = 0
\]

or, after discretizing in time:

\[
\hat{u}_j^{n+1} = \hat{u}_j^{n-1} - 2\Delta t(ick\hat{u}_j^n)
\]
Fourier-Legendre transform

$$A(\lambda_j, \phi_k, t) = \sum_m \sum_n \hat{A}_n^m(t) P_n^m(\mu_k) e^{im\lambda_j} = \sum_m \sum_n \hat{A}_n^m(t) Y_n^m(\lambda_j, \mu_k)$$

Step 1: Discrete Fourier transform $A$ along latitude circles

$$A_m(\mu_k, t) = \frac{1}{2M} \sum_{j=0}^{2M+1} A(\lambda_j, \mu_k, t) e^{-im\lambda_j} \quad \text{where} \quad \lambda_j = \frac{\pi}{M} j$$

Step 2: Legendre transform $A_m$ in the north-south direction

$$\hat{A}_n^m(t) = \sum_{k=1}^K w(\mu_k) A_m(\mu_k, t) P_n^m(\mu_k)$$

Above image from http://www.du.edu/~jcalvert/math/harmonic/harmonic.htm
Barotropic Vorticity Equation

The two-dimensional, non-divergent, inviscid, barotropic vorticity equation (BVE) states that absolute vorticity, $\eta$, is conserved following the 2D non-divergent flow:

$$\frac{D\eta}{Dt} = 0$$

Written in terms of the streamfunction, the Eulerian form of this equation is:

$$\frac{\partial \nabla^2 \psi}{\partial t} = \frac{1}{a^2 \cos \phi} \left( \frac{\partial \psi}{\partial \phi} \frac{\partial \nabla^2 \psi}{\partial \lambda} - \frac{\partial \psi}{\partial \lambda} \frac{\partial \nabla^2 \psi}{\partial \phi} \right) - \frac{2\Omega}{a^2} \frac{\partial \psi}{\partial \lambda} = F(\lambda, \mu)$$

Expressing the streamfunction and advection in terms of (triangularly truncated) spherical harmonics:

$$\psi = \sum_{m=0}^{N} \sum_{n=|m|}^{N} \hat{\psi}_n^m P_n^m(\mu)e^{im\lambda}$$

$$F = \sum_{m=0}^{N} \sum_{n=|m|}^{N} \hat{F}_n^m P_n^m(\mu)e^{im\lambda}$$
results in:

\[-\frac{n(n+1)}{a^2} \frac{d\psi_m^n}{dt} = \hat{F}_m^n\]

Discretizing in time (Leap Frog):

\[(\psi_m^n)_{\tau+1} = (\psi_m^n)_{\tau-1} - \frac{2a^2}{n(n+1)} \Delta t (\hat{F}_m^n)_\tau\]
Triangular truncation

For any spectral representation of a field, $A$, $n \leq m$. ‘$m$’ is the zonal wavenumber, ‘$n$’ is the degree of the associated Legendre polynomial, ‘$n-m$’ is like a meridional wavenumber.

\[ A(\lambda, \phi, t) = \sum_{m=0}^{N} \sum_{n=-|m|}^{N} \hat{A}_n^m(t) P_n^m(\mu) e^{im\lambda} \]
Set up

- Create your own subdirectory:
  \textit{mkdir yourlastname}

- cd into that directory
  \textit{cd yourlastname}

- Copy the code and related scripts
  \textit{cp ~dnnelson/311labs10/nelson/morgan_bve.tar .}

- Untar the code and related scripts
  \textit{tar –xvf morgan_bve.tar}

- cd into the spectral subdirectory
  \textit{cd spectral}

- Edit *.sh files to define the current directory

Code will be also available from \url{http://aurora.aos.wisc.edu/~morgan/bsm.html}
Part 1. Spectral truncation

Part 1. *Spectral decomposition (using local data)*

- *make all*
- Run `./fnl_tri_trunc.sh 060209 00 80` (will create a T80 truncation of the NCEP final analysis of relative vorticity and place results in a file t80.gem)
- Run `gdplot`; restore settings from the file trunc (i.e., *restore trunc*)
- Experiment with different resolutions – note the vorticity and streamfunction structure. *Note that each time you run* `fnl_tri_trunc.sh` *this script, a new GEMPAK file tmmm.gem will be created; as a consequence, in gdplot, you’ll have to change gdfile.*
The model

- Initial vorticity from GFS
- Initial streamfunction
- Transform streamfunction
- Calculate forcing on Gaussian grid
- Time step ahead for new streamfunction
- Output at 3 hour intervals
Part 2. Running the model

**Part 2. Barotropic spectral model (using local data)**

- Run `./fnl_bve_t108.sh 060209 00` (this will compile the BVE model and have it run out for 48 hours starting from 0000 UTC 9 February 2006). The model output is then converted into a GEMPAK grid file (`psi.gem`).
- `cd` to the model directory (`cd model`).
- The program `gdplot` may be used to view `psi.gem` (`restore bve`).
- Try to rerun the model with the data from 1200 UTC 29 August 2005.
Part 3. Running the model in near real-time

- Run `./bve_t108.sh 060209 00` (this will compile the BVE model and have it run out for 48 hours starting from 0000 UTC 9 February 2006). The model output is then converted into a GEMPAK grid file (`psi.gem`).
- `cd` to the model directory (`cd model`).
- The program `gdplot` may be used to view `psi.gem` (`restore bve`).
Part 4. Running the model using current data

• Edit bve_t108.sh by changing the data path as follows: `set data_path = /weather/data/grib`
• Run `./bve_t108.sh yymmdd hh` (this will compile the BVE model and have it run out for 48 hours starting from hh00 UTC dd mm yyyy). The model output is then converted into a GEMPAK grid file (`psi.gem`).
• `cd` to the model directory (`cd model`).
• The program `gdplot` may be used to view `psi.gem` (`restore bve`).
Part 5. Running the model using current data and comparing to actual model forecast

- Edit bve_48h_t108.sh by changing the data path as follows: `set data_path = /weather/data/grib`
- Run `./bve_48h_t108.sh yymmdd hh` (this will compile the BVE model and have it run out for 48 hours starting from hh00 UTC dd mm yyyy). The BVE model output is then converted into a GEMPAK grid file (psi.gem).
- `cd` to the model directory (`cd model`).
- Copy now.gem into the directory by typing: `cp ../now.gem`.
  (This file contains the GFS model forecast out to 48 hours.)
- Copy 2panel.set into the directory by typing: `cp ~/dnnelson/311labs10/nelson/2panel.set`.
  The settings in 2panel.set will allow you to see a side-by-side view of both the GFS and BVE model forecasts.
- The program `gdplot` may be used to view `psi.gem` (restore 2panel.set).