Hannah Barnes

Lecture Summary: April 6

We started by deriving the centered difference approximation for second order derivatives and obtained:

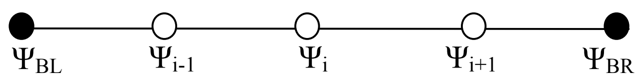
$$f''(x_o) = \frac{f(x_o + \Delta x) + f(x_0 - \Delta x) - 2f(x_o)}{\Delta x^2}$$

The rest of the lecture focused on the successive over-relaxation (SOR) method. During our lab, we will be using this method to invert a given distribution of vorticity to find the distribution of the streamfunction. This relationship is given by:

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = \zeta$$

As an example of the successive over-relaxation method, we discussed how to invert vorticity to find the streamfunction in one dimension. In other words, we assumed that $\frac{\partial^2 \Psi}{\partial y^2} = 0$.

To start we have a line of points.



The vorticity at each point is known. Additionally, we assume that the boundary conditions have a known, fixed value for the streamfunction (Ψ_{BL} and Ψ_{BR}) and guess that the interior points (Ψ_{i-1}, Ψ_i, and Ψ_{i+1}) have $\Psi = 0$.

Using the centered difference approximation for second order derivatives, the relationship between Ψ and ζ, and $\frac{\partial^2 \Psi}{\partial y^2} = 0$ we find that an updated value of Ψ can be found from:

$$\Psi_i = \frac{\Psi_{i+1} + \Psi_{i-1} - \zeta(\Delta x)^2}{2}$$

At each point, we use this equation to find Ψ_i (an updated value of Ψ for the given point) and calculate the residual (R_i) from:

$$R_i = \frac{\Psi_i - \Psi_{orginal}}{\omega}$$

where ω is the over-relaxation parameter and $\omega > 1$. ω makes R_i converge faster.

Once all the residuals are calculated, we sum them.

Prior to starting the approximation, we set a threshold for the sum of R.

If our calculated sum is less than the threshold, we know that our values of Ψ have converged and we stop. However, if our sum is larger than the threshold, it indicates that Ψ has not converged. In this case, we need to repeat the process using the updated values of Ψ at each point to find new values for Ψ_i, R_i, and the sum of R. If this sum of R is less than the threshold, we stop. However, if this sum of R is still greater than the threshold we repeat the process until the sum of R is less than the threshold.

Finally, we discussed an example Matlab code for this method.