Lecture Summary for Monday, February 16, 2009
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At the beginning of the lecture, we were reminded that the "vorticity" we had become accustomed to, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, was actually only the vertical component of the three-dimensional vorticity vector. We were further reminded that the full relative vorticity vector also has horizontal components, which take into account instances of vertical shear of the three-dimensional wind vector:

$$\vec{\omega} = \vec{\nabla} \times \vec{u} = (\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\hat{i} + (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x})\hat{j} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\hat{k}$$

The three terms of the expression for the inviscid, Lagrangian time rate of change of the absolute vorticity,

$$\frac{D}{Dt}(f + \zeta) = -(f + \zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial P}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial P}{\partial x}\right),$$

were next discussed.

The first term in the absolute vorticity equation (referred to as the ***convergence/divergence term***) was shown to describe how a pure convergence field can cause an area of positive absolute vorticity to form, due to the effects of the Coriolis Force.

In the second, bracketed term of this equation (the ***tilting/twisting term***) was shown to describe the effect of tilting a horizontal component of vorticity into the vertical by differential vertical motion. A specific example was shown that demonstrates how vertical shear of the meridional flow, $(\frac{\partial v}{\partial z})$, associated with a zonal vorticity vector can be tilted into the vertical by differential vertical motion, $\frac{\partial w}{\partial x}$. This, as Professor Morgan stated, is one of the mechanisms believed at work during tornadogenesis .

The final term (***baroclinic/solenoidal term***) describes the effect that gradients of both pressure and density have on the absolute vorticity.

Scale analysis of the vorticity equation for large-scale, atmospheric phenomena reveals that the dominant term in the absolute vorticity equation is the convergence/divergence term. Further, for such large-scale flows, the divergence (δ), and vorticity (ζ), were found to satisfy, $\delta \le O(Ro)\zeta$ and $\zeta \le O(Ro)f$, Thus, a good approximation of the Lagrangian time rate of change of the absolute vorticity (following horizontal motions) is:
$\frac{D_h}{Dt}(f + \zeta) = -f\delta$, where the divergence $\delta \le 10^{-6} s^{-1}$. This is similar to vorticity equation for of the shallow water equations discussed earlier.