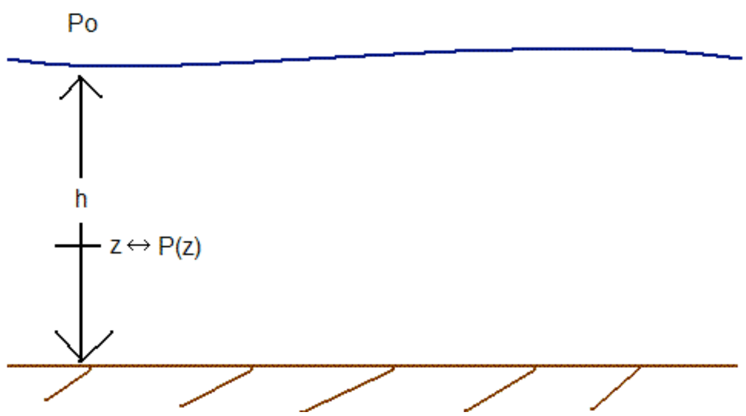
Taylor Austin

1/28/08 notes for AOS 311



P_0 is the pressure at the surface of the fluid
h is the depth of the homogeneous ($\rho = \rho_0$), rotating ($f = f_0$) fluid

An expression for the pressure at some depth z in the fluid was derived by integrating the hydrostatic equation, giving P(z) = $\mathbf{P_0 - \rho_0 g z}$ $+\rho_0 g h$. Where the bold terms are hydrostatic and the italic terms are dynamic.

By substituting in the dynamic pressure into the momentum equations you get the **Shallow Water Equations:**

A: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f\,v = -g\frac{\partial h}{\partial x}$

B: $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f\,u = -g\frac{\partial h}{\partial y}$

C: $\frac{Dh}{Dt} = -h(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$

A vorticity equation was derived by taking the curl of the horizontal momentum equations. That is, $\frac{\partial}{\partial x}A - \frac{\partial}{\partial y}B$, which simplifies to: $\frac{D}{Dt}(f + \zeta) = -\delta(f + \zeta)$.
Lagrangian increases (decreases) of vorticity are associated with horizontal convergence (divergence).

Note we call ζ the relative vorticity and f the planetary vorticity.

If we substitute $\delta = (-\frac{1}{h})(\frac{Dh}{Dt})$ into equation C, we get $\frac{D}{Dt}(\frac{f + \zeta}{h}) = 0$

The **Shallow Water Potential Vorticity:** $q_{sw} \equiv \frac{f + \zeta}{h}$.

Note q_{sw} is a conserved, dynamical variable that places a constraint on vorticity and depth changes of the fluid system - if the depth increases (decreases) there is a corresponding increase (decrease) in the vorticity.