11.) H 4.11: Given a cyclonic vortex in cyclostrophic balance with \( V = V_0 \left( \frac{r}{r_0} \right)^n \), where \( V_0 \) is the tangential velocity at distance \( r_0 \). Find the following values at distance \( r \) from the center of the vortex: circulation, vorticity, and pressure assuming that \( \rho = \rho_0 \).

**Circulation:** We know that circulation is defined as the closed line integral of the component of velocity that is tangent to the loop, \( C = \oint u \cdot dl \).

In this case, at fixed radius, the tangential velocity is a constant \( V \), and
\[
C = \oint u \cdot dl = V \oint dl = 2\pi V.
\]
As a consequence, \( C = 2\pi V = 2\pi V_0 \frac{r^{n+1}}{r_0^n} \).

**Vorticity:** \( \zeta = \frac{\partial V}{\partial r} + \frac{V}{r} = \frac{1}{r} \frac{\partial}{\partial r} (rV) = \frac{V_0}{r_0^n r} \frac{\partial}{\partial r} (r^{n+1}) = (n+1) \frac{V_0}{r_0^n} r^{n-1} = (n+1) \frac{V}{r} \)

**Pressure:** For cyclostrophic flow, the pressure gradient force balances the centrifugal force, so we get:
\[
\frac{V^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial r}.
\]
Integrating this yields:
\[
\int_{p_0}^{p} \frac{dp}{p} = \int \frac{V_0^2 r^{2n-1}}{r_0^{2n}} dr.
\]
Integrating both sides, gives:
\[
\frac{p - p_0}{\rho} = \frac{V_0^2}{r_0^{2n}} \left[ \frac{r^{2n} - r_0^{2n}}{2n} \right] = \frac{V^2 - V_0^2}{2n}.
\]