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4 February 2009 Lecture Summary

First we defined the *circulation (C)* over a closed chain of fluid parcels where **u** is the 3-dimensional velocity vector and **dl** is the segment of the chain. $C = \oint \boldsymbol{u} \cdot \boldsymbol{dl}$ We defined the counterclockwise direction to be positive. By Stoke's theorem we equated this expression for circulation to the double line integral of the curl times area. $C = \oint \boldsymbol{u} \cdot \boldsymbol{dl} = \oiint (\nabla x \cdot \boldsymbol{u}) \cdot dA$ And we determined that Circulation divided by Area is the average vorticity, $\bar{\zeta}$. Then we did an example of a rectangular closed chain of fluid parcels and found that Circulation is equal to vorticity for a small enough dA.

$$\lim_{dA \to 0} \frac{C}{dA} = \zeta\ (vorticity)$$

Next we took a look at the irrotational vortex where $u_\Theta = \frac{B}{2\pi r}$. We did two examples where we found the circulation for two different areas of this vortex. The first was an area not including the origin and we found that the circulation in this region was zero. Then we calculated the circulation of an area including the origin and found that it was "B". We found that for any circulation around the origin, the circulation equals "B". We determined "B" to be a measure of the strength of the vortex. So we determined that for a point vortex, the vorticity is concentrated at the center, and we compared that feature to the electric fields of particles.

Then we developed the (absolute) circulation equation which describes the Lagrangian time tendency of absolute circulation: $\frac{DC}{Dt} = \oint -\frac{dp}{\rho} + \oint \boldsymbol{F} \cdot dl$