AOS 311 Lecture 8 “Circulation”

**Brief description**

In this lecture we defined the circulation about a closed loop of fluid parcels and related it to the momentum equations...T..., in a homogeneous fluid... was conserved.

New terminology

**circulation:** the line integral of the tangential component of the flow about a closed loop of fluid parcels.

**baroclinic fluid:** a fluid within which the density is a function of both pressure and temperature.

Topics covered

1. The circulation, $C$, about a closed chain of fluid parcels is the closed line integral of the velocity tangent to the chain (loop), i.e., $C = \oint (\mathbf{v} \cdot d\mathbf{t})$. Application of Stokes' Theorem to the definition of circulation yields: $C = \oint (\nabla \times \mathbf{u}) \cdot dA$. This form of the definition of circulation relates the circulation about a closed chain of fluid parcels to the area integral of the vorticity enclosed by that loop. From this form, we may loosely say that “circulation is vorticity ‘times’ area.”

2. A circulation equation, expressing the Lagrangian time rate of change of absolute circulation, $\frac{DC}{Dt}$, was derived for a frame of reference fixed with respect to the stars:

   $$\frac{DC}{Dt} = \frac{D}{Dt} (\oint \mathbf{u} \cdot d\mathbf{t}) = \oint \frac{Du}{Dt} \cdot d\mathbf{t} + \oint [\oint \mathbf{u} \cdot \frac{D}{Dt} (d\mathbf{t})] = \oint \frac{Du}{Dt} \cdot d\mathbf{t} + \oint \mathbf{u} \cdot d\mathbf{u} = \oint \frac{Du}{Dt} \cdot d\mathbf{t} + \oint \frac{\mathbf{d}l}{2}$$

3. Substituting for Lagrangian acceleration, the three fundamental forces acting on a fluid parcel, we obtain: $\frac{DC}{Dt} = \oint \frac{Du}{Dt} \cdot d\mathbf{t} + \oint \mathbf{F}_f \cdot d\mathbf{t}$, where $\mathbf{F}_f$ is frictional force. Simplifying this expression gives: $\frac{DC}{Dt} = \oint \mathbf{F}_f \cdot d\mathbf{t}$

4. For inviscid flows, $\frac{DC}{Dt} = \oint \frac{dp}{\rho}$, generation or destruction of circulation is governed by the solenoidal term (baroclinic generation/destruction term). For inviscid, barotropic flows (for which homogenous flows are a special case), the absolute circulation is conserved. For homogeneous flows, $\frac{DC}{Dt} = \oint \frac{dp}{\rho} = \oint \frac{dp}{\rho_0} = -\frac{1}{\rho_0} \oint dp = 0$. The last equality is valid because $dp$ is an exact differential.

Reading: H: Chapters 1-3; M: Chapters 1-4; C-R: Chapters 1-3

H 4.1 (for this lecture)