AOS 311 Lecture 7 “Vorticity Dynamics”

**Brief description**
In this lecture, we derived and interpreted the vorticity equation for the shallow water system. We discovered that vorticity is a conserved dynamical variable of the shallow water system.

**New terminology**
- **Planetary vorticity**: represented as \( f \) in the shallow water and potential vorticity equations, it is comparable to the Coriolis force for momentum equations.
- **Potential vorticity**: represented as \( q \), is dependent on local variables, so it is characteristic of the parcel.

**Topics Covered**

1. Derived the shallow water vorticity equation by taking the curl of the horizontal momentum equations and assuming constant \( f \):
   \[
   \frac{D}{Dt} \zeta + (f + \zeta) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0.
   \]
   Since the Lagrangian derivative of \( f \) is zero:
   \[
   \frac{Df}{Dt} = 0,
   \]
   we can add it to the first term on the left hand side of the above equation to yield the final from of our shallow water vorticity equation:
   \[
   \frac{D}{Dt} (f + \zeta) + (f + \zeta)\delta = 0, \quad \text{where } \delta \text{ is the horizontal divergence.}
   \]

2. The fact that \( f \) is added to the (relative) vorticity \( \zeta \) in the vorticity equation suggests that it too is a form of vorticity. We may refer to \( f \) as the “planetary vorticity.”

3. It was demonstrated that in the presence of cyclonic vorticity, horizontal convergence acts to increase the cyclonic vorticity.

4. The shallow water potential vorticity (PV) equation was derived by multiplying the vorticity equation by \( h \) and subtracting from it \((f + \zeta)\) times the mass continuity equation:
   \[
   (f + \zeta) \frac{Dh}{Dt} = -h(f + \zeta)\delta,
   \]
   yields:
   \[
   h \frac{D(f + \zeta)}{Dt} - (f + \zeta) \frac{Dh}{Dt} = 0.
   \]
   Dividing both sides by \( h^2 \), yields:
   \[
   \frac{D(f + \zeta)}{Dt} = 0.
   \]
   We define shallow water PV \((q_{sw})\) as:
   \[
   q_{sw} = \frac{f + \zeta}{h}.
   \]
   The shallow water PV equation expresses the fact that \( q_{sw} \) is conserved following the fluid motion.

5. Scale analysis of the ratio of relative vorticity to planetary vorticity, for geostrophic flows reveals,
   \[
   \frac{\zeta}{f} \sim \frac{U}{fL} \sim R_e << 1.
   \]
   In this scaling we assumed:
   \[
   \zeta \sim \frac{U}{L}, f \sim f_e.
   \]
   We can then say:
   \[
   q_{sw} = \frac{f}{h}.
   \]
   For constant \( f \), the geostrophic flow must be along isobaths (and isobars).

Reading: C-R: Ch 4; M-P: Ch 6-7
C-R: Ch 4-4 (for this lecture)