AOS 311 Lecture 7 “Vorticity Dynamics”

Brief description
In this lecture, we determined vorticity equations for shallow water systems and explored the balance between vorticity and the coriolis force. We discussed the importance of shallow water potential vorticity in fluid dynamics so we can apply it later for large-scale cyclonic motions.

New terminology
Planetary vorticity: represented as $f$ in the shallow water and potential vorticity equations. Comparable to the coriolis force for momentum equations.

Potential vorticity: represented as $q$, is dependent on local variables, so it is characteristic of the parcel.

Topics Covered
1. Derived the shallow water vorticity equation by taking the curl of the horizontal and vertical momentum equations, combining them together, and substituting relative vorticity to get:
$$\frac{D}{Dt} \zeta + (f + \zeta) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0.$$ Since the lagrangian derivative of $f$: $\frac{Df}{Dt}=0$, we can add it to the above equation and combine the lagrangian forms of $f$ and $\zeta$, substitute horizontal divergence to yield our final shallow water vorticity equation:
$$\frac{D}{Dt} (f + \zeta) + (f + \zeta)\delta = 0.$$

2. Discussed conservation of momentum with respect to divergence and vorticity. For an initially horizontally convergent parcel, cyclonic sheared vorticity is generated or strengthened by the planetary vorticity acting on the parcel. As the distance between two parallel cyclonic wind vectors decreases, shear increases. Convergence acting on those two vectors increases cyclonic vorticity.

3. Derived shallow water potential vorticity equation and proved conservation of mass by applying the mass continuity equation. Multiplying our vorticity equation by $h$ and subtracting the mass continuity equation: $(f + \zeta) \frac{Dh}{Dt} = -h(f + \zeta)\delta$, yields:
$$h \frac{D(f + \zeta)}{Dt} - (f + \zeta) \frac{Dh}{Dt} = 0.$$ Dividing both sides by $h^2$ and integrating yields: $\frac{D}{Dt} (f + \zeta) = 0$, so we define shallow water potential vorticity:
$$q_{sw} = \frac{f + \zeta}{h},$$ where the lagrangian derivative is conserved.

4. Scale analysis for shallow water systems: $\zeta \sim \frac{U}{L} f \sim f_o$, $\frac{\zeta}{f} = \frac{U}{f_o L} = \frac{U}{fL} \sim R_o << 1$, implying geostrophic balance when $f$ dominates. We can then say: $q \approx \frac{f}{h}$ for rapidly rotating flows. For constant $f$, the flow has to be along isobaths.

Reading: C-R: Ch 4; M-P: Ch 6-7
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