AOS 311 Lecture 4 "Boussinesq Approximation"

Brief description
In this lecture, we applied the Boussinesq approximation to the equations of motion and the mass continuity equation. A scale analysis was performed on the reduced form of the vertical momentum equation.

New terminology
Boussinesq approximation: an approximation to the governing fluid dynamics equations in which variations in density are neglected except where gravity multiplies density (i.e., in the vertical momentum equation).

dynamic pressure: that portion of the three-dimensional pressure field that is responsible for horizontal accelerations and associated motion.

Topics covered
1. Introduced the concept of the Boussinesq approximation
2. After using the approximation, the simplified momentum equations become:
   a. zonal: $\frac{Du}{Dt} + f_* w - f v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \nabla^2 u$, where $\nu = \mu / \rho_0$.
   b. vertical: $\frac{Dw}{Dt} - f_* u = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} - g - \frac{\rho'}{\rho_0} g + \nu \nabla^2 w$, where $\frac{\rho'}{\rho_0} g$ is the buoyancy force - an important force when considering stability and convection.
3. The vertical momentum equation (VME) was further simplified by partitioning the total pressure, $p(x,y,z,t)$, into a hydrostatic pressure, $p_0(z)$, and dynamic pressure, $p'(x,y,z,t)$, i.e., $p = p_0 + p'$, where $\partial p_0 / \partial z = -\rho_0 g$. After substituting $p_0 + p'$ in for pressure in the VME and simplifying, we get the reduced form: $\frac{Dw}{Dt} - f_* u = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} - \frac{\rho'}{\rho_0} g + \nu \nabla^2 w$. 
4. A scale analysis was performed on the reduced form of the VME. By assuming that the flow is shallow ($H << L$), and $Ro, Ro_T, << 1$, we determined that the terms within $\frac{Dw}{Dt}$ and the Coriolis force term are small and can be neglected relative to the buoyancy term, $-g \frac{\rho'}{\rho_0}$. Assuming $Ek << 1$, the last term on the right hand side of the equation is neglected and the final vertical momentum equation becomes: $\frac{1}{\rho_0} \frac{\partial p'}{\partial z} = -\frac{\rho'}{\rho_0} g$. This means that even dynamic (i.e., non-static) flows are hydrostatic.

Reading: H: Chapters 1-3; M: Chapters 1-4; C-R: Chapters 1-3
C-R (3:3-4 for this lecture)