AOS 311 Lecture 30 “Kelvin Waves”

**Brief description**
In this lecture, we continued to look at Kelvin waves with the corresponding assumptions and equations. We then introduced planetary and topographic Rossby waves.

**New terminology**

**Topics covered**

1. We looked at a semi-infinite fluid with a wall of height H on one side. We then assumed an f plane, where \( f = f_o \), and \( u' = 0 \) everywhere in the semi-infinite space.
   a. After subbing in \( u' = 0 \) to the momentum equations, the equations then were
   \[
   -f v' = -g \frac{\partial h'}{\partial x}, \quad \frac{\partial v'}{\partial t} = -g \frac{\partial h'}{\partial y} + H(\frac{\partial v'}{\partial y}) = 0.
   \]
   Because we assumed \( u' = 0 \) and because the fluid is semi-infinite, then \( v', h' \) must be functions of \( x \) as follows:
   \[
   v' = \hat{v}(x)e^{i(\beta y - \omega t)} \quad \text{and} \quad h' = \hat{h}(x)e^{i(\beta y - \omega t)}.
   \]
   Solving for \( v' \), you get
   \[
   v' = \frac{g \hat{h}}{f} \frac{\partial}{\partial x}.
   \]
   After substitution of the functions, we found that the dispersion relationship is
   \[
   \frac{\omega}{L} = c_r = \pm \sqrt{gH},
   \]
   which is the phase speed. From this we see that the waves only propagate in the north to south direction, or along the coast.
   b. We then set two equations of \( v' \) equal to one another and found that
   \[
   \frac{\partial h}{\partial x} = \frac{\hat{h}}{L_r},
   \]
   where \( L_r = \frac{c_r}{f} \), which is the Rossby radius of deformation. The N-S moving waves create piling up of water with the boundary to the right of the flow. The flow is in geostrophic balance in the x direction and converging in the y direction.

2. Then we looked at waves where changes in \( f \) are important, called planetary and topographic Rossby waves. In these waves we are seeking near geostrophic motion, where \( \omega \) is small.
   a. Assume \( \beta \)-plane approximation to the momentum equations:
   \[
   \frac{\partial u'}{\partial t} - (f_o + \beta_y y) v' = -g \frac{\partial h'}{\partial x}, \quad \frac{\partial v'}{\partial t} + (f_o + \beta_y y) u' = -g \frac{\partial h'}{\partial y} + H(\frac{\partial v'}{\partial y}) = 0.
   \]
   We then applied the geostrophic flows for \( v' \) and ended with
   \[
   v' = \frac{g}{f_o} \frac{\partial h'}{\partial x} - \frac{g}{f_o} \beta_y \frac{\partial (\frac{\partial h'}{\partial t})}{\partial y} \frac{\partial}{\partial t} + \frac{\beta_y g \frac{\partial h'}{\partial x}}{f_o^2}.
   \]
   Where \( \frac{g}{f_o} \frac{\partial h'}{\partial x} \) is geostrophic balance and \( \frac{g}{f_o} \frac{\partial}{\partial t} + \frac{\beta_y g \frac{\partial h'}{\partial x}}{f_o^2} \) is the ageostrophic correction, called the isallobaric wind.

Reading: C-R 6 (6.4-6.5 for this lecture)