AOS 311 Lecture 16 “The 2D, inviscid, non-divergent, barotropic vorticity equation”

**Brief description**
In this lecture, we investigated the dynamics described by the two-dimensional, inviscid, non-divergent, barotropic, vorticity equation. We derived and interpreted its local (Eulerian) derivative form and wrote the equation in terms of streamfunction, $\psi$.  

**New terminology**

**$\beta$-plane approximation:** An approximation in which the Coriolis parameter, $f$, is treated as a constant except where it is differentiated with respect to $y$: $f = f_0 + \beta \beta_y$.  

**$f$-plane approximation:** An approximation in which the Coriolis parameter, $f$, is treated as a constant: $f = f_0$.

**Topics covered**

1. For barotropic, inviscid, non-divergent, 2-D flow, absolute vorticity is conserved: \[ \frac{D(f + \zeta)}{Dt} = 0. \]

2. To evaluate the local tendency of vorticity, we write the total (Lagrangian) derivative as the sum of the Eulerian (local) derivative and the advective derivative of absolute vorticity:  
   \[ \frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial x} - v \frac{\partial \zeta}{\partial y} - \nu \beta, \]  
   where $\beta \equiv \frac{df}{dy}$ is the meridional gradient of planetary vorticity. The term on the left hand side of the equation is the local tendency in relative vorticity. The first and second terms on the right hand side are the zonal and meridional advections of relative vorticity, respectively. The final term is the meridional advection of planetary vorticity.

3. We see from this equation that the local change in vorticity is due solely to the horizontal advection of vorticity.

4. Since we have a 2-D non-divergent flow, the flow may be described by a streamfunction, $\psi$:  
   \[ \mathbf{V}_\psi = \mathbf{k} \times \nabla \psi \]  
   (where $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$) and the relative vorticity given by:  
   \[ \zeta = \nabla^2 \psi. \]

5. Rewriting the Eulerian form of the barotropic vorticity equation in terms of $\psi$ yields:  
   \[ \frac{\partial \nabla^2 \psi}{\partial t} = \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} \nabla^2 \psi - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \nabla^2 \psi - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial x} \beta = F(\psi), \]  
   where $F(\psi)$, the forcing term, a function of streamfunction alone, represents the advection of absolute vorticity by the 2-D, non-divergent flow.

6. Finally, we examined how we might predict the flow evolution using this last equation. Given a an initial distribution of vorticity $\zeta$, we may invert the relation $\zeta = \nabla^2 \psi$ to solve for the streamfunction, and compute the forcing, $F(\psi)$. We then step ahead an arbitrary time increment (the smaller the increment the more accurate, but also more computationally time costly) and the cycle continues. This method has proved to make a useful model for some short-term forecasts.

Reading: H: Chapter 4; M: Chapter 5  
H: Chapter 4.5 (for this lecture)