AOS 311 Lecture 15 "Vorticity Equation"

**Brief description**

In this lecture, we discussed the tilting/twisting term in the vorticity equation. We then compared the vorticity equation to the shallow water system equations, and found that the shallow water potential vorticity equation with a rigid lid at a fixed depth reduces to a statement of conservation of absolute vorticity.

**New terminology**

**Topics covered**

1. When a vorticity maximum is collocated with maximum stretching, any air parcel in that area **experiences an increase in absolute vorticity.**

2. Vorticity is a vector! \( \mathbf{\omega} = \nabla \times \mathbf{u} = (\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}) \mathbf{i} - (\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}) \mathbf{j} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) \mathbf{k} \). We are most often concerned with the vertical component as it is perpendicular to weather maps - the "swirl" of circulation is in the same plane as the map (x-y plane).

3. The tilting/twisting term \( -\frac{\partial w}{\partial x} \frac{\partial u}{\partial y} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \) describes how vorticity in the x-y plane is tilted into the vertical.
   a. In the tilting/twisting term, the vertical shears of the zonal and meridional wind \( \frac{\partial u}{\partial z} \) and \( \frac{\partial v}{\partial z} \) respectively are associated with (positive) components of vorticity in the meridional and zonal directions.
   b. The differential vertical motion terms \( \frac{\partial v}{\partial x} \) and \( \frac{\partial v}{\partial y} \) describe the tilting process.

4. The term \( f \nabla \times \mathbf{u} \) in the vorticity equation that dominates for large-scale flows is the convergence/divergence term. The term that dominates for mesoscale processes is the tilting/twisting term and the baroclinic generation term can be important for mesoscale phenomena (phenomena with horizontal length scales of less than about 300 km including severe thunderstorms and tornadoes, and organized convective complexes).

5. We considered the conservation of shallow water potential vorticity:
\[
q_{sw} = \frac{f + \zeta}{h} = \text{const.
}
\]

For a constant depth, \((h = \text{const.})\), absolute vorticity, \(\eta\), is conserved:
\[
\frac{D}{Dt} (f + \zeta) = 0
\]

6. In the mid-troposphere, vertical motions are maximized at the level of nondivergence. At this level, typically found at about 500 hPa, for barotropic, frictionless, two-dimensional flows, the absolute vorticity, \(\eta\), is conserved.

Reading: H: Chapter 4; M: Chapter 5
H: Chapter 4.4 and M: Chapter 5.3 (for this lecture)