In this lecture, we continued discussing Kelvin’s Circulation Theorem by deriving the portion of absolute circulation ($C_a$) that results from Earth’s rotation about its axis ($C_e$), and discussing the factors that can change relative circulation with respect to time.

New terminology

**planetary vorticity**: the local vertical component of the vorticity of a fluid parcel that exists due to the Earth’s rotation.

**Topics Covered**

1. We considered a closed area of barotropic, inviscid fluid with no motion relative to the earth, whose sides are as shown to the right.

2. We calculated $C_e = \oint u \cdot dl$ where $u_e$ is the velocity the parcel would have if it were fixed to the earth. We determined $u_e = \Omega \cos\phi$ and $dl_1 = [d|l|] = a \cos\phi dl$, where $dl$ is the distance change in longitude, $u_e = \Omega \cos(\phi + d\phi)$.

3. We found $C_e = \oint u \cdot dl = u_{13} \cdot dl_1 + u_{13} \cdot dl_2 = \Omega a^2 \cos(\phi - \cos^2(\phi + d\phi))dl$. We then divided $C_e$ by $A$ and took the limit as $d\phi$ goes to 0 (requiring the use of l’Hôpital’s rule) to leave us with $\frac{C_e}{A} = 2\Omega \sin\phi$, the value of the Coriolis parameter $f$. Because we can roughly say “circulation = vorticity • area”, we know $f = 2\Omega \sin\phi$ must represent a vorticity, which we call the planetary vorticity.

4. We may write $C_e = 2\Omega <\sin\phi > A = 2\Omega A$, where $A$ is the projection of the parcel onto an equatorial plane and the brackets indicate an average with respect to latitude.

5. Plugging $C_e$ back into Kelvin’s Circulation Theorem ($C_a = C_e + C_r$), taking the derivative with respect to time, and solving for the derivative of relative circulation $\frac{DC_r}{Dt} = \frac{DC}{Dt}$ gave us $\frac{DC_r}{Dt} = -\oint \frac{d\rho}{\rho} \cdot F \cdot dl - 2\Omega \frac{DA}{Dt}$. This shows us that for a barotropic, inviscid fluid, changes in circulation are associated with changes in $A$, through:
   - $a$. changes in latitude or
   - $b$. changes in area.

An increase (decrease) in either will cause a decrease (increase) in $C$.

6. We considered an example that showed the effects on the average tangential flow around a fluid parcel as it changed its latitude.

Reading: H: Ch 4.1-4.3; M: Ch 5.1-5.2
H: 4.1, M: 5.1 (for this lecture)