AOS 311 Lecture 10 “Kelvin’s Circulation Theorem”

Brief description

In this lecture, we continued discussing Kelvin’s Circulation Theorem by first deriving the component of absolute circulation ($C_a$) that results from Earth’s rotation about its axis ($C_e$), discussing the factors that can change absolute circulation over time, and doing an example.

New terminology

planetary vorticity: the local vertical component of the vorticity of the earth due to its rotation.

Topics covered

1. We considered a closed area of barotropic, inviscid fluid with no motion relative to the earth and sides as shown at right.

2. We calculated $C_e = \int u_e \cdot dl$, where $u_e$ is the velocity the parcel would have if it were fixed to the earth. We determined $u_i = \Omega a \cos \phi$, $dl_1 = a \cos \phi d\lambda$, where $d\lambda$ is the distance change in longitude, $u_z = \Omega a \cos(\phi + d\phi)$, $dl_3 = a \cos(\phi + d\phi) d\lambda$, and the area of the parcel to be $A = \frac{1}{2} ad\phi (dl_1 + dl_3)$.

3. We found $C_e = \int u_e \cdot dl = u_{e1} \cdot dl_1 + u_{e3} \cdot dl_3 = \Omega a^2 \cos^2 \phi - \cos^2(\phi + d\phi) d\lambda$. We then divided $C_e$ by $A$ and took the limit as $d\phi$ goes to 0 (requiring the use of l’Hôpital’s rule) to leave us with $\frac{C_e}{A} = 2\Omega \sin \phi$, the value of the Coriolis parameter $f$. Because we can roughly say “circulation = vorticity · area”, we know $f = 2\Omega \sin \phi$ must represent a vorticity, which we call the planetary vorticity.

4. To simplify calculations, we can approximate $C_e = 2\Omega < \sin \phi > A = 2\Omega A_e$ where $A_e$ is the projection of the parcel onto an equatorial plane.

5. Plugging $C_e$ back into Kelvin’s Circulation Theorem ($C_a = C_e + C_i$), taking the derivative with respect to time, and solving for the derivative of relative circulation ($\frac{DC}{Dt}$) gave us

$$\frac{DC}{Dt} = \int \frac{dp}{\rho} + \int F_i \cdot dl - 2\Omega \frac{DA_e}{Dt}.$$ This shows us that for a barotropic, inviscid fluid, changes in circulation will be caused by any change in $A_e$ through:

- a change in latitude or
- a change in area.
- An increase in either will cause a decrease in $C$.

6. We considered an example that showed the effects on average tangential flow around a fluid parcel due to changing its latitude.